

## Linear Regression

### EXERCISE 3.1 [PAGES 41 - 42]

#### Exercise 3.1 | Q 1 | Page 41

The HRD manager of a company wants to find a measure which he can use to fix the monthly income of persons applying for the job in the production department. As an experimental project, he collected data of 7 persons from that department referring to years of service and their monthly incomes.

<b>Years of service (X)</b>	11	7	9	5	8	6	10
<b>Monthly Income (₹ 1000's)(Y)</b>	10	8	9	5	9	7	11

- Find the regression equation of income on years of service.
- What initial start would you recommend for a person applying for the job after having served in a similar capacity in another company for 13 years?

**Solution:** (i) Here,  $X$  = Years of service,

$Y$  = Income (₹ in 1000's)

$X = x_i$	$Y = y_i$	$x_i^2$	$x_i y_i$
11	10	121	110
7	8	49	56
9	6	81	54
5	5	25	25
8	9	64	72
6	7	36	42
10	11	100	110
<b>56</b>	<b>56</b>	<b>476</b>	<b>469</b>

From the table, we have



$$n = 7, \sum x_i = 56, \sum y_i = 56, \sum x_i^2 = 476,$$

$$\sum x_i y_i = 469$$

$$\bar{x} = \sum \frac{x_i}{n} = \frac{56}{7} = 8$$

$$\bar{y} = \sum \frac{y_i}{n} = \frac{56}{7} = 8$$

$$\text{Now, } b_{YX} = \frac{\sum x_i y_i - n \bar{x} \bar{y}}{\sum x_i^2 - n \bar{x}^2}$$

$$\therefore b_{YX} = \frac{469 - 7 \times 8 \times 8}{476 - 7 \times (8)^2} = \frac{469 - 448}{476 - 448} = \frac{21}{28} = \frac{3}{4}$$

$$\therefore b_{YX} = 0.75$$

$$\text{Also, } a = \bar{y} - b_{YX} \bar{x} = 8 - 0.75 \times 8 = 8 - 6 = 2$$

$\therefore$  The regression equation of income (Y) on years of service (X) is

$$Y = a + b_{YX} X$$

$$\therefore Y = 2 + 0.75 X$$

(ii) Given, years of service (X) = 13

$\therefore$  Substituting X = 13 in regression equation, we get

$$Y = 2 + 0.75 (13)$$

$$\therefore Y = 2 + 9.75$$

$$\therefore Y = 11.75 (\text{₹ in 1000's})$$

$\therefore$  I would recommend an initial start of ₹ 11,750 for a person who has served in similar capacity in another company for 13 years.

### Exercise 3.1 | Q 2 | Page 41

Calculate the regression equations of X on Y and Y on X from the following data:

<b>X</b>	10	12	13	17	18
<b>Y</b>	5	6	7	9	13

**Solution:**

<b>X = x<sub>i</sub></b>	<b>Y = y<sub>i</sub></b>	<b>x<sub>i</sub><sup>2</sup></b>	<b>y<sub>i</sub><sup>2</sup></b>	<b>x<sub>i</sub> y<sub>i</sub></b>
10	5	100	25	50
12	6	144	36	72
13	7	169	49	91
17	9	289	81	153
18	13	324	169	234
<b>70</b>	<b>40</b>	<b>1026</b>	<b>360</b>	<b>600</b>

From the table, we have,

$$n = 5, \sum x_i = 70, \sum y_i = 40, \sum x_i y_i = 600, \sum x_i^2 = 1026,$$

$$\sum y_i^2 = 360$$

$$\bar{x} = \frac{\sum x_i}{n} = \frac{70}{5} = 14,$$

$$\bar{y} = \frac{\sum y_i}{n} = \frac{40}{5} = 8$$

Now, for regression equation of X on Y

$$b_{XY} = \frac{\sum x_i y_i - n \bar{x} \bar{y}}{\sum y_i^2 - n \bar{y}^2}$$

$$= \frac{600 - 5 \times 14 \times 8}{360 - 5(8)^2} = \frac{600 - 560}{360 - 320} = \frac{40}{40} = 1$$

$$\text{Also, } a' = \bar{x} - b_{XY} \bar{y} = 14 - 1(8) = 14 - 8 = 6$$

∴ The regression equation of X on Y is

$$X = a' + b_{XY}Y$$

$$∴ X = 6 + Y$$

Now, for regression equation of Y on X

$$\begin{aligned} b_{YX} &= \frac{\sum x_i y_i - n \bar{x} \bar{y}}{\sum x_i^2 - n \bar{x}^2} \\ &= \frac{600 - 5(14)(8)}{1026 - 5(14)^2} = \frac{600 - 560}{1026 - 980} = \frac{40}{46} = 0.87 \end{aligned}$$

$$\text{Also, } a = \bar{y} - b_{YX} \bar{x}$$

$$= 8 - 0.87 \times 14 = 8 - 12.18 = -4.18$$

∴ The regression equation of Y on X is

$$Y = a + b_{YX} X$$

$$∴ Y = -4.18 + 0.87X$$

### Exercise 3.1 | Q 3 | Page 41

For a certain bivariate data on 5 pairs of observations given

$$\sum x = 20, \sum y = 20, \sum x^2 = 90, \sum y^2 = 90, \sum xy = 76$$

Calculate:

- i.  $\text{cov}(X, Y)$
- ii.  $b_{YX}$  and  $b_{XY}$
- iii.  $r$

**Solution:** Given,  $\sum x = 20, \sum y = 20, \sum x^2 = 90, \sum y^2 = 90, \sum xy = 76, n = 5$

Now,

$$\bar{x} = \frac{\sum x}{n} = \frac{20}{5} = 4$$

$$\bar{y} = \frac{\sum y}{n} = \frac{20}{5} = 4$$

$$(i) \text{ cov } (X, Y) = \frac{1}{n} \sum xy - \bar{x} \bar{y}$$

$$= \frac{1}{5} \times 76 - 4 \times 4$$

$$= 15.2 - 16 = -0.8$$

$$(ii) b_{YX} = \frac{\sum xy - n\bar{x}\bar{y}}{\sum x^2 - n\bar{x}^2}$$

$$= \frac{76 - 5 \times 4 \times 4}{90 - 5(4)^2} = \frac{76 - 80}{90 - 80} = \frac{-4}{10} = -0.4$$

$$b_{YX} = \frac{\sum xy - n\bar{x}\bar{y}}{\sum y^2 - n\bar{y}^2}$$

$$= \frac{76 - 5 \times 4 \times 4}{90 - 5(4)^2} = \frac{76 - 80}{90 - 80} = -0.4$$

$$(iii) r = \pm \sqrt{b_{XY} \cdot b_{YX}}$$

$$= \pm \sqrt{(-0.4)(-0.4)} = \pm \sqrt{0.16} = \pm 0.4$$

Since  $b_{YX}$  and  $b_{XY}$  both are negative,

$r$  is negative.

$$\therefore r = -0.4$$

**Exercise 3.1 | Q 4 | Page 41**

From the following data estimate y when x = 125.

<b>X</b>	120	115	120	125	126	123
<b>Y</b>	13	15	14	13	12	14

**Solution:** In order to estimate y, we have to find the regression equation of Y on X.

<b>X = <math>x_i</math></b>	<b>Y = <math>y_i</math></b>	<b><math>x_i^2</math></b>	<b><math>x_i y_i</math></b>
120	13	14400	1560
115	15	13225	1725
120	14	14400	1680
125	13	15625	1625
126	12	15876	1512
123	14	15129	1722
<b>729</b>	<b>81</b>	<b>88655</b>	<b>9824</b>

From table, we get

$$n = 6, \sum x_i = 729, \sum y_i = 81, \sum x_i^2 = 88655, \sum x_i y_i = 9824$$

$$\therefore \bar{x} = \frac{\sum x_i}{n} = \frac{729}{6} = 121.5$$

$$\therefore \bar{y} = \frac{\sum y_i}{n} = \frac{81}{6} = 13.5$$

Now,

$$b_{YX} = \frac{\sum x_i y_i - n \bar{x} \bar{y}}{\sum x_i^2 - n \bar{x}^2}$$



$$= \frac{9824 - 6 \times 121.5 \times 13.5}{88655 - 6(121.5)^2}$$

$$= \frac{9824 - 9841.5}{88655 - 88573.5} = \frac{-17.5}{81.5} = -0.22$$

Also,  $a = \bar{y} - b_{YX} \bar{x}$

$$= 13.5 - (-0.22) \times 121.5$$

$$= 13.5 + 26.73 = 40.23$$

$\therefore$  The regression equation of Y on X is

$$Y = a + b_{YX} X$$

$$\therefore Y = 40.23 - 0.22 X$$

For  $X = 125$ ,

$$Y = 40.23 - 0.22 \times 125 = 40.23 - 27.5 = 12.73$$

$\therefore$  The estimated value of Y is 12.73 for  $X = 125$ .

### Exercise 3.1 | Q 5.1 | Page 41

The following table gives the aptitude test scores and productivity indices of 10 workers selected at random.

<b>Aptitude score (X)</b>	60	62	65	70	72	48	53	73	65	82
<b>Productivity Index (Y)</b>	68	60	62	80	85	40	52	62	60	81

Obtain the two regression equations and estimate the productivity index of a worker whose test score is 95.

**Solution:** Here,  $X$  = Aptitude score,  $Y$  = Productivity index

$X = x_i$	$Y = y_i$	$x_i - \bar{x}$	$y_i - \bar{y}$	$(x_i - \bar{x})^2$	$(y_i - \bar{y})^2$	$(x_i - \bar{x})(y_i - \bar{y})$
60	68	-5	3	25	9	-15
62	60	-3	-5	9	25	15
65	62	0	-3	0	9	0
70	80	5	15	25	225	75
72	85	7	20	49	400	140
48	40	-17	-25	289	625	425
53	52	-12	-13	144	169	156
73	62	8	-3	64	9	-24
65	60	0	-5	0	25	0
82	81	17	16	289	256	272
<b>650</b>	<b>650</b>	-	-	<b>894</b>	<b>1752</b>	<b>1044</b>

From the table, we have

$$n = 10, \sum x_i = 650, \sum y_i = 650$$

$$\therefore \bar{x} = \frac{\sum x_i}{n} = \frac{650}{10} = 65$$

$$\bar{y} = \frac{\sum y_i}{n} = \frac{650}{10} = 65$$

Since the mean of X and Y are whole numbers, we will use the formula

$$b_{YX} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2} \quad \text{and} \quad b_{XY} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (y_i - \bar{y})^2}$$

From the table, we have



$$\sum (x_i - \bar{x})(y_i - \bar{y}) = 1044, \sum (x_i - \bar{x})^2 = 894, \sum (y_i - \bar{y}) =$$

$$b_{YX} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2} = \frac{1044}{894} = 1.16$$

$$\text{Now, } a = \bar{y} - b_{YX}\bar{x}$$

$$= 65 - 1.16 \times 65 = 65 - 75.4 = -10.4$$

∴ The regression equation of productivity index (Y) on Aptitude score (X) is

$$Y = a + b_{YX} X$$

$$\therefore Y = -10.4 + 1.16 X$$

For X = 95,

$$Y = -10.4 + 1.16(95) = -10.4 + 110.2 = 99.8$$

∴ The productivity index of worker with a test score of 95 is 99.8.

### Exercise 3.1 | Q 5.2 | Page 41

The following table gives the aptitude test scores and productivity indices of 10 workers selected at random.

<b>Aptitude score (X)</b>	60	62	65	70	72	48	53	73	65	82
<b>Productivity Index (Y)</b>	68	60	62	80	85	40	52	62	60	81

Obtain the two regression equations and estimate the test score when the productivity index is 75.

**Solution:** Here, X = Aptitude score, Y = Productivity index

$X = x_i$	$Y = y_i$	$x_i - \bar{x}$	$y_i - \bar{y}$	$(x_i - \bar{x})^2$	$(y_i - \bar{y})^2$	$(x_i - \bar{x})(y_i - \bar{y})$
60	68	-5	3	25	9	-15
62	60	-3	-5	9	25	15
65	62	0	-3	0	9	0
70	80	5	15	25	225	75
72	85	7	20	49	400	140
48	40	-17	-25	289	625	425
53	52	-12	-13	144	169	156
73	62	8	-3	64	9	-24
65	60	0	-5	0	25	0
82	81	17	16	289	256	272
<b>650</b>	<b>650</b>	-	-	<b>894</b>	<b>1752</b>	<b>1044</b>

From the table, we have

$$n = 10, \sum x_i = 650, \sum y_i = 650$$

$$\therefore \bar{x} = \frac{\sum x_i}{n} = \frac{650}{10} = 65$$

$$\bar{y} = \frac{\sum y_i}{n} = \frac{650}{10} = 65$$

Since the mean of X and Y are whole numbers, we will use the formula

$$b_{YX} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2} \text{ and } b_{XY} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (y_i - \bar{y})^2}$$

From the table, we have

$$\sum (x_i - \bar{x})(y_i - \bar{y}) = 1044, \sum (x_i - \bar{x})^2 = 894, \sum (y_i - \bar{y})^2 = 1752$$

$$b_{XY} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (y_i - \bar{y})^2} = \frac{1044}{1752} = 0.59$$

$$\text{Now, } a' = \bar{x} - b_{XY}\bar{y}$$

$$= 65 - 0.59 \times 65 = 65 - 38.35 = 26.65$$



∴ The regression equation of Aptitude score (X) on productivity index (Y) is

$$X = a' + b_{XY} Y$$

$$\therefore X = 26.65 + 0.59 Y$$

For Y = 75,

$$X = 26.65 + 0.59 \times 75 = 26.65 + 44.25 = 70.9$$

∴ The test score is 70.9 when productivity index is 75

### Exercise 3.1 | Q 6 | Page 42

Compute the appropriate regression equation for the following data:

X [Independent Variable]	2	4	5	6	8	11
Y [dependent Variable]	18	12	10	8	7	5

**Solution:** Since X is independent and Y is dependent variable, we find the regression equation of Y on X.

$X = x_i$	$Y = y_i$	$x_i^2$	$x_i y_i$
2	18	4	36
4	12	16	48
5	10	25	50
6	8	36	48
8	7	64	56
11	5	121	55
<b>36</b>	<b>60</b>	<b>266</b>	<b>293</b>



From the table, we have,

$$n = 6, \sum x_i = 36, \sum y_i = 60, \sum x_i^2 = 266, \sum x_i y_i = 293$$

$$\bar{x} = \frac{\sum x_i}{n} = \frac{36}{6} = 6$$

$$\bar{y} = \frac{\sum y_i}{n} = \frac{60}{6} = 10$$

$$\begin{aligned} \text{Now, } b_{YX} &= \frac{\sum x_i y_i - n \bar{x} \bar{y}}{\sum x_i^2 - n \bar{x}^2} \\ &= \frac{293 - 6 \times 6 \times 10}{266 - 6(6)^2} = \frac{293 - 360}{266 - 216} = \frac{-67}{50} \end{aligned}$$

$$\therefore b_{YX} = -1.34$$

$$\text{Also, } a' = \bar{y} - b_{YX} \bar{x}$$

$$= 10 - (-1.34)(6) = 10 + 8.04 = 18.04$$

The regression equation of Y on X is

$$Y = a + b_{YX} X$$

$$\therefore Y = 18.04 - 1.34 X$$

### Exercise 3.1 | Q 7 | Page 42

The following are the marks obtained by the students in Economics (X) and Mathematics (Y)

X	59	60	61	62	63
Y	78	82	82	79	81

Find the regression equation of Y on X.

**Solution:** X = Marks obtained in Economics

Y = Marks obtained in Mathematics

<b>X = x<sub>i</sub></b>	<b>Y = y<sub>i</sub></b>	<b>x<sub>i</sub><sup>2</sup></b>	<b>x<sub>i</sub> y<sub>i</sub></b>
59	78	3481	4602
60	82	3600	4920
61	82	3721	5002
62	79	3844	4898
63	81	3969	5103
<b>305</b>	<b>402</b>	<b>18615</b>	<b>24525</b>

From the table, we have

$$n = 6, \sum x_i = 305, \sum y_i = 402, \sum x_i^2 = 18615, \sum x_i y_i = 24525$$

$$\bar{x} = \frac{\sum x_i}{n} = \frac{305}{5} = 61$$

$$\bar{y} = \frac{\sum y_i}{n} = \frac{402}{5} = 80.4$$

$$\begin{aligned} \text{Now, } b_{YX} &= \frac{\sum x_i y_i - n \bar{x} \bar{y}}{\sum x_i^2 - n \bar{x}^2} \\ &= \frac{24525 - 5 \times 61 \times 80.4}{18615 - 5(61)^2} \\ &= \frac{24525 - 24522}{18615 - 18605} = \frac{3}{10} = 0.3 \end{aligned}$$

$$\begin{aligned} \text{Also, } a &= \bar{y} - b_{YX} \bar{x} \\ &= 80.4 - (0.3)(61) = 80.4 - 18.3 = 62.1 \end{aligned}$$

The regression equation of Y on X is

$$Y = a + b_{YX} X$$

$$\therefore Y = 62.1 + 0.3 X$$

### Exercise 3.1 | Q 8 | Page 42

For the following bivariate data obtain the equations of two regression lines:

<b>X</b>	1	2	3	4	5
<b>Y</b>	5	7	9	11	13

**Solution:**

<b>X = x<sub>i</sub></b>	<b>Y = y<sub>i</sub></b>	<b>x<sub>i</sub><sup>2</sup></b>	<b>y<sub>i</sub><sup>2</sup></b>	<b>x<sub>i</sub> y<sub>i</sub></b>
1	5	1	25	5
2	7	4	49	14
3	9	9	81	27
4	11	16	121	44
5	13	25	169	65
<b>15</b>	<b>45</b>	<b>55</b>	<b>445</b>	<b>155</b>

From the table, we have

$$n = 6, \sum x_i = 15, \sum y_i = 45, \sum x_i^2 = 55, \sum y_i^2 = 445, \sum x_i y_i = 155$$

$$\bar{x} = \frac{\sum x_i}{n} = \frac{15}{5} = 3$$

$$\bar{y} = \frac{\sum y_i}{n} = \frac{45}{5} = 9$$

Now, for regression equation of Y on X,

$$\begin{aligned} b_{YX} &= \frac{\sum x_i y_i - n\bar{x}\bar{y}}{\sum x_i^2 - n\bar{x}^2} \\ &= \frac{155 - 5 \times 3 \times 9}{55 - 5(3)^2} = \frac{155 - 135}{55 - 45} = \frac{20}{10} = 2 \end{aligned}$$

$$\text{Also, } a = \bar{y} - b_{XY} \bar{x} = 9 - 2(3) = 9 - 6 = 3$$

The regression analysis of Y on X is

$$Y = a + b_{YX} X$$

$$\therefore Y = 3 + 2X$$

Now, for regression equation of X on Y,

$$\begin{aligned} b_{XY} &= \frac{\sum x_i y_i - n\bar{x}\bar{y}}{\sum y_i^2 - n\bar{y}^2} \\ &= \frac{155 - 5 \times 3 \times 9}{445 - 5(9)^2} = \frac{155 - 135}{445 - 405} = \frac{20}{40} = 0.5 \end{aligned}$$

$$\text{Also, } a' = \bar{x} - b_{XY} \bar{y}$$

$$= 3 - (0.5)(9) = 3 - 4.5 = -1.5$$

The regression equation of X on Y is

$$X = a' + b_{XY} Y$$

$$\therefore X = -1.5 + 0.5Y$$

### Exercise 3.1 | Q 9 | Page 42

From the following data obtain the equation of two regression lines:

X	6	2	10	4	8
Y	9	11	5	8	7

Solution:

$X = x_i$	$Y = y_i$	$x_i^2$	$y_i^2$	$x_i y_i$
6	9	36	81	54
2	11	4	121	22
10	5	100	25	50
4	8	16	64	32
8	7	64	49	56
<b>30</b>	<b>40</b>	<b>220</b>	<b>340</b>	<b>214</b>

From the table, we have

$$n = 6, \sum x_i = 30, \sum y_i = 40, \sum x_i^2 = 220, \sum y_i^2 = 340, \sum x_i y_i = 214$$

$$\bar{x} = \frac{\sum x_i}{n} = \frac{30}{6} = 5$$

$$\bar{y} = \frac{\sum y_i}{n} = \frac{40}{6} = 6.67$$

Now, for regression equation of Y on X,

$$b_{YX} = \frac{\sum x_i y_i - n \bar{x} \bar{y}}{\sum x_i^2 - n \bar{x}^2}$$





$$= \frac{214 - 5 \times 6 \times 8}{220 - 5(6)^2} = \frac{214 - 240}{220 - 180} = \frac{-26}{40} = -0.65$$

Also,  $a = \bar{y} - b_{YX} \bar{x}$

$$= 8 - (-0.65)(6) = 8 + 3.9 = 11.9$$

The regression equation of Y on X is

$$Y = a + b_{YX} X$$

$$\therefore Y = 11.9 - 0.65X$$

Now, for regression equation of X on Y,

$$b_{XY} = \frac{\sum x_i y_i - n \bar{x} \bar{y}}{\sum y_i^2 - n \bar{y}^2}$$

$$= \frac{214 - 5 \times 6 \times 8}{340 - 5(8)^2} = \frac{214 - 240}{340 - 320} = \frac{-26}{20} = -1.3$$

Also,  $a' = \bar{x} - b_{XY} \bar{y}$

$$= 6 - (-1.3)8 = 6 + 10.4 = 16.4$$

The regression equation of Y on X is

$$X = a' + b_{XY} Y$$

$$\therefore X = 16.4 - 1.3Y$$

### Exercise 3.1 | Q 10 | Page 42

For the following data, find the regression line of Y on X

X	1	2	3
Y	2	1	6

Hence find the most likely value of y when x = 4.

**Solution:**

<b>X = x<sub>i</sub></b>	<b>Y = y<sub>i</sub></b>	<b>x<sub>i</sub><sup>2</sup></b>	<b>x<sub>i</sub> y<sub>i</sub></b>
1	2	1	2
2	1	4	2
3	6	9	18
<b>6</b>	<b>9</b>	<b>14</b>	<b>22</b>

From the table, we have

$$n = 3, \sum x_i = 6, \sum y_i = 9, \sum x_i^2 = 14, \sum x_i y_i = 22$$

$$\bar{x} = \frac{\sum x_i}{n} = \frac{6}{3} = 2$$

$$\bar{y} = \frac{\sum y_i}{n} = \frac{9}{3} = 3$$

$$\begin{aligned}\text{Now, } b_{YX} &= \frac{\sum x_i y_i - n \bar{x} \bar{y}}{\sum x_i^2 - n \bar{x}^2} \\ &= \frac{22 - 3 \times 2 \times 3}{14 - 3(2)^2} = \frac{22 - 18}{14 - 12} = \frac{4}{2} = 2\end{aligned}$$

$$\text{Also, } a = \bar{y} - b_{YX} \bar{x} = 3 - 2(2) = -1$$

The regression equation of Y on X is,

$$Y = a + b_{YX} X$$

$$\therefore Y = -1 + 2X$$

For X = 4,

$$Y = -1 + 2(4) = -1 + 8 = 7$$

The most likely value of Y for X = 4 is 7.

**Exercise 3.1 | Q 11 | Page 42**

From the following data, find the regression equation of Y on X and estimate Y when X = 10.

<b>X</b>	1	2	3	4	5	6
<b>Y</b>	2	4	7	6	5	6

**Solution:**

<b>X = <math>x_i</math></b>	<b>Y = <math>y_i</math></b>	<b><math>x_i^2</math></b>	<b><math>x_i y_i</math></b>
1	2	1	2
2	4	4	8
3	7	9	21
4	6	16	24
5	5	25	25
6	6	36	36
<b>21</b>	<b>30</b>	<b>91</b>	<b>116</b>

From the table, we have

$$n = 6, \sum x_i = 21, \sum y_i = 30, \sum x_i^2 = 91, \sum x_i y_i = 116$$

$$\bar{x} = \frac{\sum x_i}{n} = \frac{21}{6} = 3.5$$

$$\bar{y} = \frac{\sum y_i}{n} = \frac{30}{6} = 5$$



$$\text{Now, } b_{YX} = \frac{\sum x_i y_i - n \bar{x} \bar{y}}{\sum x_i^2 - n \bar{x}^2}$$

$$= \frac{116 - 6 \times 3.5 \times 5}{91 - 6(3.5)^2} = \frac{116 - 105}{91 - 73.5} = \frac{11}{17.5} = 0.63$$

$$\text{Also, } a = \bar{y} - b_{YX} \bar{x}$$

$$= 5 - 0.63 \times 3.5$$

$$= 5 - 2.205 = 2.8$$

The regression equation of Y on X is,

$$Y = a + b_{YX} X$$

$$\therefore Y = 2.8 + 0.63 X$$

For  $X = 10$ ,

$$Y = 2.8 + 0.63 \times 10$$

$$= 2.8 + 6.3 = 9.1$$

$\therefore$  The value of Y when  $X = 10$  is 9.1

### Exercise 3.1 | Q 12 | Page 42

The following sample gives the number of hours of study (X) per day for an examination and marks (Y) obtained by 12 students.

X	3	3	3	4	4	5	5	5	6	6	7	8
Y	45	60	55	60	75	70	80	75	90	80	75	85

Obtain the line of regression of marks on hours of study.

**Solution:**

X = $x_i$	Y = $y_i$	$x_i^2$	$x_i y_i$
3	45	9	135

3	60	9	180
3	55	9	165
4	60	16	240
4	75	16	300
5	70	25	350
5	80	25	400
5	75	25	375
6	90	36	540
6	80	36	480
7	75	49	525
8	85	64	680
<b>59</b>	<b>850</b>	<b>319</b>	<b>4370</b>

From the table, we have

$$n = 12, \sum x_i = 59, \sum y_i = 850, \sum x_i^2 = 319, \sum x_i y_i = 4370$$

$$\bar{x} = \frac{\sum x_i}{n} = \frac{59}{12} = 4.92$$

$$\bar{y} = \frac{\sum y_i}{n} = \frac{850}{12} = 70.83$$

$$\begin{aligned} \text{Now, } b_{YX} &= \frac{\sum x_i y_i - n \bar{x} \bar{y}}{\sum x_i^2 - n \bar{x}^2} \\ &= \frac{4370 - 12 \times 4.92 \times 70.83}{319 - 12 \times (4.92)^2} \\ &= \frac{4370 - 4181.80}{319 - 290.48} \\ &= \frac{188.2}{28.52} = 6.6 \end{aligned}$$

$$\text{Also, } a = \bar{y} - b_{YX} \bar{x}$$

$$= 70.83 - 6.6 \times 4.92$$

$$= 70.83 - 32.47$$

$$= 38.36$$

∴ The regression equation of Y on X is,

$$Y = a + b_{YX} X$$

$$∴ Y = 38.36 + 6.6 X$$

### EXERCISE 3.2 [PAGES 47 - 48]

#### Exercise 3.2 | Q 1.1 | Page 47

For bivariate data.  $\bar{x} = 53, \bar{y} = 28, b_{YX} = -1.2, b_{XY} = -0.3$

Find Correlation coefficient between X and Y.

**Solution:**

Here,  $\bar{x} = 53, \bar{y} = 28, b_{YX} = -1.2, b_{XY} = -0.3$

$$r = \pm \sqrt{b_{XY} \cdot b_{YX}}$$

$$= \pm \sqrt{(-0.3)(-1.2)}$$

$$= \pm \sqrt{0.36} = \pm 0.6$$

Since  $b_{YX}$  and  $b_{XY}$  both are – negative,

r is also negative.

$$∴ r = - 0.6$$

#### Exercise 3.2 | Q 1.2 | Page 47

For bivariate data.  $\bar{x} = 53, \bar{y} = 28, b_{YX} = -1.2, b_{XY} = -0.3$

Find estimate of Y for X = 50.

**Solution:**

Here,  $\bar{x} = 53$ ,  $\bar{y} = 28$ ,  $b_{YX} = -1.2$ ,  $b_{XY} = -0.3$

The regression equation of Y on X is

$$(Y - \bar{y}) = b_{YX}(X - \bar{x})$$

$$\therefore (Y - 28) = (-1.2)(X - 53)$$

$$\therefore Y - 28 = -1.2X + 63.6$$

$$\therefore Y = -1.2X + 63.6 + 28$$

$$\therefore Y = -1.2X + 91.6$$

For  $X = 50$

$$\therefore Y = -1.2(50) + 91.6 = -60 + 91.6 = 31.6$$

### Exercise 3.2 | Q 1.3 | Page 47

For bivariate data.  $\bar{x} = 53$ ,  $\bar{y} = 28$ ,  $b_{YX} = -1.2$ ,  $b_{XY} = -0.3$

Find estimate of X for  $Y = 25$ .

**Solution:**

Here,  $\bar{x} = 53$ ,  $\bar{y} = 28$ ,  $b_{YX} = -1.2$ ,  $b_{XY} = -0.3$

The regression equation of X on Y is

$$(X - \bar{x}) = b_{XY}(Y - \bar{y})$$

$$\therefore (X - 53) = (-0.3)(Y - 28)$$

$$\therefore X - 53 = -0.3Y + 8.4$$

$$\therefore X = -0.3Y + 8.4 + 53$$

$$\therefore X = -0.3Y + 61.4$$

For  $Y = 25$

$$\therefore X = -0.3(25) + 61.4 = -7.5 + 61.4 = 53.9$$

### Exercise 3.2 | Q 2 | Page 47

From the data of 20 pairs of observations on X and Y, following results are obtained.

$$\bar{x} = 199, \bar{y} = 94,$$

$$\sum (x_i - \bar{x})^2 = 1200, \sum (y_i - \bar{y})^2 = 300,$$

$$\sum (x_i - \bar{x})(y_i - \bar{y}) = -250$$

Find:

- The line of regression of Y on X.
- The line of regression of X on Y.
- Correlation coefficient between X and Y.

**Solution:**

$$\text{Given, } n = 20, \bar{x} = 199, \bar{y} = 94,$$

$$\sum (x_i - \bar{x})^2 = 1200, \sum (y_i - \bar{y})^2 = 300,$$

$$\sum (x_i - \bar{x})(y_i - \bar{y}) = -250$$

$$(i) b_{YX} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2} = \frac{-250}{1200} = -\frac{5}{24}$$

$\therefore$  The regression equation of Y on X is

$$(Y - \bar{y}) = b_{YX}(X - \bar{x})$$

$$\therefore (Y - 94) = -\frac{5}{24}(X - 199)$$

$$\therefore 24Y - 2256 = -5X + 995$$

$$\therefore 5X + 24Y = 3251$$



$$(ii) b_{YX} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (y_i - \bar{y})^2} = \frac{-250}{300} = -\frac{5}{6}$$

∴ The regression equation of X on Y is

$$(X - \bar{x}) = b_{XY}(Y - \bar{y})$$

$$\therefore (X - 199) = -\frac{5}{6} (Y - 94)$$

$$\therefore 6X - 1194 = -5Y + 470$$

$$\therefore 6X + 5Y = 1664$$

$$(iii) r = \pm \sqrt{b_{XY} \cdot b_{YX}}$$

$$= \pm \sqrt{\left(-\frac{5}{24}\right) \left(-\frac{5}{6}\right)} = \pm \sqrt{\frac{25}{144}} = \pm \frac{5}{12}$$

Since  $b_{XY}$  and  $b_{YX}$  both are negative,

$r$  is also negative.

$$\therefore r = -5/12$$

### Exercise 3.2 | Q 3 | Page 47

From the data of 7 pairs of observations on X and Y, following results are obtained.

$$\sum (x_i - 70) = -35, \quad \sum (y_i - 60) = -7,$$

$$\sum (x_i - 70)^2 = 2989, \quad \sum (y_i - 60)^2 = 476,$$

$$\sum (x_i - 70)(y_i - 60) = 1064$$

$$[\text{Given: } \sqrt{0.7884} = 0.8879]$$

Obtain

- i. The line of regression of Y on X.
- ii. The line regression of X on Y.
- iii. The correlation coefficient between X and Y.

**Solution:** Given:  $n = 7$ ,  $\sum(x_i - 70) = -35$ ,  $\sum(y_i - 60) = -7$ ,

$$\sum(x_i - 70)^2 = 2989, \quad \sum(y_i - 60)^2 = 476,$$

$$\sum(x_i - 70)(y_i - 60) = 1064$$

$$\text{Let } u_i = x_i - 70 \text{ and } v_i = y_i - 60$$

$$\therefore \sum u_i = -35, \quad \sum v_i = -7$$

$$\sum u_i^2 = 2989, \quad \sum v_i^2 = 479$$

$$\sum u_i v_i = 1064$$

$$\therefore \bar{u} = \frac{\sum u_i}{n} = \frac{-35}{7} = -5$$

$$\therefore \bar{v} = \frac{\sum v_i}{n} = \frac{-7}{7} = -1$$

$$\begin{aligned} \text{Now, } \sigma_u^2 &= \frac{\sum u_i^2}{n} - (\bar{u})^2 \\ &= \frac{2989}{7} - (-5)^2 = 427 - 25 = 402 \end{aligned}$$

$$\begin{aligned} \text{and } \sigma_v^2 &= \frac{\sum v_i^2}{n} - (\bar{v})^2 \\ &= \frac{476}{7} - (-1)^2 = 68 - 1 = 67 \end{aligned}$$

$$\begin{aligned}\text{cov}(u, v) &= \frac{\sum u_i v_i}{n} - \bar{u}\bar{v} \\ &= \frac{1064}{7} - (-5)(-1) = 152 - 5 = 147\end{aligned}$$

Since the regression coefficients are independent of change of origin,

$$b_{YX} = b_{VU} \text{ and } b_{XY} = b_{UV}$$

$$\therefore b_{YX} = b_{VU} = \frac{\text{cov}(u, v)}{\sigma_U^2} = \frac{147}{402} = 0.36$$

$$\text{and } b_{XY} = b_{UV} = \frac{\text{cov}(u, v)}{\sigma_V^2} = \frac{147}{67} = 2.19$$

$$\text{Also, } \bar{x} = \bar{u} + 70 = -5 + 70 = 65$$

$$\text{and } \bar{y} = \bar{v} + 60 = -1 + 60 = 59$$

**(i)** The line of regression of Y on X is

$$(Y - \bar{y}) = b_{YX}(X - \bar{x})$$

$$\therefore (Y - 59) = (0.36)(X - 65)$$

$$\therefore Y - 59 = 0.36X - 23.4$$

$$\therefore Y = 0.36X + 59 - 23.4$$

$$\therefore Y = 0.36X + 35.6$$

**(ii)** The line of regression of X on Y is

$$(X - \bar{x}) = b_{XY}(Y - \bar{y})$$

$$\therefore (X - 65) = (2.19)(Y - 59)$$

$$\therefore X - 65 = 2.19Y - 129.21$$

$$\therefore X = 2.19Y + 65 - 129.21$$

$$\therefore X = 2.19Y - 64.21$$

$$\begin{aligned}
 \text{(iii) } r &= \pm \sqrt{b_{YX} \cdot b_{XY}} \\
 &= \pm \sqrt{(0.36)(2.19)} \\
 &= \pm \sqrt{0.7884} = \pm 0.8879
 \end{aligned}$$

Since  $b_{YX}$  and  $b_{XY}$  both are positive,

$r$  is also positive.

$$\therefore r = 0.8879$$

### Exercise 3.2 | Q 4 | Page 47

You are given the following information about advertising expenditure and sales.

	Advertisement expenditure (₹ in lakh) (X)	Sales (₹ in lakh) (Y)
Arithmetic Mean	10	90
Standard Mean	3	12

Correlation coefficient between X and Y is 0.8

- Obtain the two regression equations.
- What is the likely sales when the advertising budget is ₹ 15 lakh?
- What should be the advertising budget if the company wants to attain sales target of ₹ 120 lakh?

**Solution:**

$$\text{Given: } \bar{x} = 10, \bar{y} = 90, \sigma_x = 3, \sigma_y = 12, r = 0.8$$

$$b_{YX} = r \frac{\sigma_y}{\sigma_x} = 0.8 \times \frac{12}{3} = 0.8 \times 4 = 3.2$$

$$b_{XY} = r \frac{\sigma_x}{\sigma_y} = 0.8 \times \frac{3}{12} = 0.8 \times 0.25 = 0.2$$

(i) The regression equation of Y on X is

$$(Y - \bar{y}) = b_{YX}(X - \bar{x})$$

$$\therefore (Y - 90) = 3.2 (X - 10)$$

$$\therefore Y - 90 = 3.2 X - 32$$

$$\therefore Y = 3.2 X - 32 + 90$$

$$\therefore Y = 3.2 X + 58 \quad \dots(i)$$

The regression equation of X on Y is

$$(X - \bar{x}) = b_{XY}(Y - \bar{y})$$

$$\therefore (X - 10) = 0.2 (Y - 90)$$

$$\therefore X - 10 = 0.2 Y - 18$$

$$\therefore X = 0.2 Y - 18 + 10$$

$$\therefore X = 0.2 Y - 8 \quad \dots(ii)$$

(ii) For  $X = 15$ , from equation (i) we get

$$Y = 3.2 (15) + 58 = 48 + 58 = 106$$

$\therefore$  Likely sales is ₹ 106 lakh when advertising budget is ₹ 15 lakh.

(iii) For  $Y = 120$ , from equation (ii) we get

$$X = 0.2 (120) - 8 = 24 - 8 = 16$$

$\therefore$  To attain sales target of ₹ 120 lakh, advertising budget must be ₹ 16 lakh.

### Exercise 3.2 | Q 5.1 | Page 47

**Bring out the inconsistency in the following:**

$$b_{YX} + b_{XY} = 1.30 \text{ and } r = 0.75$$

**Solution:** Given,  $b_{YX} + b_{XY} = 1.30$ ,  $r = 0.75$

Consider,  $\frac{b_{YX} + b_{XY}}{2} = \frac{1.30}{2} = 0.65$

$\therefore \frac{b_{YX} + b_{XY}}{2} < r$

But, for consistent data  $\left| \frac{b_{YX} + b_{XY}}{2} \right| > |r|$

$\therefore$  Given data is inconsistent.

### Exercise 3.2 | Q 5.2 | Page 47

**Bring out the inconsistency in the following:**

$b_{YX} = b_{XY} = 1.50$  and  $r = -0.9$

**Solution:** Given,  $b_{YX} = b_{XY} = 1.50$  and  $r = -0.9$

Here, the coefficient of regressions is positive and the coefficient of correlation is negative.

But, for consistent data they must have the same signs.

$\therefore$  The given data is inconsistent.

### Exercise 3.2 | Q 5.3 | Page 47

**Bring out the inconsistency in the following:**

$b_{YX} = 1.9$  and  $b_{XY} = -0.25$

**Solution:** Given,  $b_{YX} = 1.9$  and  $b_{XY} = -0.25$

Here,  $b_{YX}$  and  $b_{XY}$  have different signs.

But, for consistent data, they must have the same signs.

$\therefore$  The given data is inconsistent.

### Exercise 3.2 | Q 5.4 | Page 47

**Bring out the inconsistency in the following:**

$b_{YX} = 2.6$  and  $b_{XY} = 1/2.6$

**Solution:** Given,  $b_{YX} = 2.6$  and  $b_{XY} = 1/2.6$

Here,  $b_{YX}$  and  $b_{XY}$  have the same signs.

Also,  $b_{YX} > 1$  and  $b_{XY} < 1$

Also, for consistent data, the signs of  $b_{YX}$  and  $b_{XY}$  are same and  $b_{YX} > 1$ ,  $b_{XY} < 1$

Here,  $b_{YX} \cdot b_{XY} = 1$

$\therefore$  The given data is consistent.

### Exercise 3.2 | Q 6 | Page 47

Two samples from bivariate populations have 15 observations each. The sample means of X and Y are 25 and 18 respectively. The corresponding sum of squares of deviations from respective means is 136 and 150. The sum of the product of deviations from respective means is 123. Obtain the equation of the line of regression of X on Y.

**Solution:**

Given,  $n = 15$ ,  $\bar{x} = 25$ ,  $\bar{y} = 18$ ,

$$\sum (x_i - \bar{x})^2 = 136, \sum (y_i - \bar{y})^2 = 150,$$

$$\sum (x_i - \bar{x})(y_i - \bar{y}) = 123$$

$$\text{Now, } b_{XY} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (y_i - \bar{y})^2} = \frac{123}{150} = 0.82$$

$$\text{Also, } a' = \bar{x} - b_{XY} \bar{y}$$

$$= 25 - 0.82 \times 18 = 25 - 14.76 = 10.24$$

$\therefore$  The regression equation of X on Y is

$$X = a' + b_{XY} Y$$

$$\therefore X = 10.24 + 0.82Y$$

### Exercise 3.2 | Q 7 | Page 48

For a certain bivariate data

	X	Y
Mean	25	20
S.D.	4	3

And  $r = 0.5$ . Estimate y when  $x = 10$  and estimate x when  $y = 16$

**Solution:**



Given,  $\bar{x} = 25$ ,  $\bar{y} = 20$ ,  $\sigma_X = 4$ ,  $\sigma_Y = 3$ ,  $r = 0.5$

$$b_{YX} = r \frac{\sigma_y}{\sigma_x} = (0.5) \frac{3}{4} = 0.375$$

$$b_{XY} = r \frac{\sigma_y}{\sigma_x} = (0.5) \frac{4}{3} = 0.667$$

The regression equation of Y on X is

$$(Y - \bar{y}) = b_{YX}(X - \bar{x})$$

$$(Y - 20) = 0.375 (X - 25)$$

$$Y - 20 = - 9.375 + 0.375 X$$

$$Y = 10.625 + 0.375 X$$

For  $X = 10$

$$Y = 10.625 + 0.375 \times 10 = 10.625 + 3.75 = 14.375$$

The regression equation of X on Y is



$$(Y - \bar{y}) = b_{YX}(X - \bar{x})$$

$$(Y - 20) = 0.375 (X - 25)$$

$$Y - 20 = - 9.375 + 0.375 X$$

$$Y = 10.625 + 0.375 X$$

For  $X = 10$

$$Y = 10.625 + 0.375 \times 10 = 10.625 + 3.75 = 14.375$$

The regression equation of X on Y is

$$(X - \bar{x}) = b_{XY}(Y - \bar{y})$$

$$(X - 25) = 0.667(Y - 20)$$

$$X - 25 = - 13.34 + 0.667 Y$$

$$X = 11.66 + 0.667 Y$$

For  $Y = 16$ ,

$$X = 11.66 + 0.667(16) = 11.66 + 10.672 = 22.332$$

[**Note:** Answer in the textbook is incorrect.]

### Exercise 3.2 | Q 8 | Page 48

Given the following information about the production and demand of a commodity obtain the two regression lines:

	X	Y
<b>Mean</b>	85	90
<b>S.D.</b>	5	6

The coefficient of correlation between X and Y is 0.6. Also estimate the production when demand is 100.

**Solution:**

Given,  $\bar{x} = 85, \bar{y} = 90, \sigma_X = 5, \sigma_Y = 6, r = 0.6$

$$b_{YX} = r \frac{\sigma_Y}{\sigma_X} = 0.6 \times \frac{6}{5} = 0.72$$

$$b_{XY} = r \frac{\sigma_X}{\sigma_Y} = 0.6 \times \frac{5}{6} = 0.5$$

The regression equation of Y on X is

$$(Y - \bar{y}) = b_{YX}(X - \bar{x})$$

$$(Y - 90) = 0.72(X - 85)$$

$$Y - 90 = 0.72X - 61.2$$

$$Y = 0.72X - 61.2 + 90$$

$$Y = 28.8 + 0.72X \quad \dots(i)$$

The regression equation of X on Y is

$$(X - \bar{x}) = b_{XY}(Y - \bar{y})$$

$$(X - 85) = 0.5(Y - 90)$$

$$X - 85 = 0.5Y - 45$$

$$X = 0.5Y - 45 + 85$$

$$X = 40 + 0.5Y \quad \dots(ii)$$

For  $Y = 100$ , from equation (ii) we get

$$X = 40 + 0.5(100) = 40 + 50 = 90$$

$\therefore$  The production is 90 when demand is 100.

### Exercise 3.2 | Q 9 | Page 48

Given the following data, obtain a linear regression estimate of X for Y =

$$10, \bar{x} = 7.6, \bar{y} = 14.8, \sigma_x = 3.2, \sigma_y = 16 \text{ and } r = 0.7$$

**Solution:**

Given,  $\bar{x} = 7.6$ ,  $\bar{y} = 14.8$ ,  $\sigma_x = 3.2$ ,  $\sigma_y = 16$  and  $r = 0.7$

$$b_{XY} = r \frac{\sigma_X}{\sigma_Y} = 0.7 \times \frac{3.2}{16} = 0.14$$

$$a' = \bar{x} - b_{XY} \bar{y}$$

$$= 7.6 - 0.14 \times 14.8 = 7.6 - 2.072 = 5.528$$

The regression equation of X on Y is

$$X = a' + b_{XY} Y$$

$$\therefore X = 5.528 + 0.14 Y$$

For  $Y = 10$

$$X = 5.528 + 0.14 \times 10 = 5.528 + 1.4 = 6.928$$

### Exercise 3.2 | Q 10 | Page 48

An inquiry of 50 families to study the relationship between expenditure on accommodation (₹ x) and expenditure on food and entertainment (₹ y) gave the following results:

$$\sum x = 8500, \sum y = 9600, \sigma_x = 60, \sigma_y = 20, r = 0.6$$

Estimate the expenditure on food and entertainment when expenditure on accommodation is Rs 200.

**Solution:** X = Expenditure on accommodation.

Y = Expenditure on food and entertainment

$$\text{Given, } \sum x = 8500, \sum y = 9600, \sigma_x = 60, \sigma_y = 20, r = 0.6, n = 50$$

$$\therefore \bar{x} = \frac{\sum x}{n} = \frac{8500}{50} = 170$$

$$\bar{y} = \frac{\sum y}{n} = \frac{9600}{50} = 192$$

$$\text{Now, } b_{YX} = r \frac{\sigma_Y}{\sigma_X} = 0.6 \times \frac{20}{60} = 0.2$$

$$\text{Also, } a = \bar{y} - b_{YX} \bar{x}$$

$$= 192 - 0.2 \times 170 = 192 - 34 = 158$$

The regression equation of Y on X is

$$Y = a + b_{YX} X$$

$$\therefore Y = 158 + 0.2 X$$

For  $X = 200$ ,

$$Y = 158 + 0.2 \times 200 = 158 + 40 = 198$$

$\therefore$  The expenditure on food and entertainment is ₹ 198 when expenditure on accommodation is ₹ 200.

[**Note:** Answer in the textbook is incorrect.]

### Exercise 3.2 | Q 11.1 | Page 48

The following data about the sales and advertisement expenditure of a firms is given below (in ₹ Crores)

	Sales	Adv. Exp.
Mean	40	6
S.D.	10	1.5

Coefficient of correlation between sales and advertisement expenditure is 0.9.

Estimate the likely sales for a proposed advertisement expenditure of ₹ 10 crores.

**Solution:** Let  $X = \text{Sales}$ ,

Y = Advertisement expenditure

Given,  $\bar{x} = 40, \bar{y} = 6, \sigma_X = 10, \sigma_Y = 1.5, r = 0.9$

$$b_{XY} = r \frac{\sigma_X}{\sigma_Y} = 0.9 \times \frac{10}{1.5} = 6$$

$$b_{YX} = r \frac{\sigma_Y}{\sigma_X} = 0.9 \times \frac{1.5}{10} = 0.135$$

The regression equation of X on Y is

$$(X - \bar{x}) = b_{XY}(Y - \bar{y})$$

$$\therefore (X - 40) = 6(Y - 6)$$

$$\therefore X - 40 = 6Y - 36$$

$$\therefore X = 6Y - 36 + 40$$

$$\therefore X = 6Y + 4$$

For Y = 10, we get

$$X = 6(10) + 4 = 60 + 4 = 64$$

$\therefore$  The likely sale is ₹ crores for a proposed advertisement expenditure of ₹ 10 crores.

### Exercise 3.2 | Q 11.2 | Page 48

The following data about the sales and advertisement expenditure of a firms is given below (in ₹ Crores)

	Sales	Adv. Exp.
Mean	40	6
S.D.	10	1.5

Coefficient of correlation between sales and advertisement expenditure is 0.9.

What should be the advertisement expenditure if the firm proposes a sales target ₹ 60 crores?

**Solution:** Let X = Sales,

Y = Advertisement expenditure

Given,  $\bar{x} = 40, \bar{y} = 6, \sigma_X = 10, \sigma_Y = 1.5, r = 0.9$

$$b_{XY} = r \frac{\sigma_X}{\sigma_Y} = 0.9 \times \frac{10}{1.5} = 6$$

$$b_{YX} = r \frac{\sigma_Y}{\sigma_X} = 0.9 \times \frac{1.5}{10} = 0.135$$

The regression equation of Y on X is

$$(Y - \bar{y}) = b_{YX}(X - \bar{x})$$

$$\therefore (Y - 6) = 0.135(X - 40)$$

$$\therefore Y - 6 = 0.135X - 5.4$$

$$\therefore Y = 0.135X - 5.4 + 6$$

$$\therefore Y = 0.135X + 0.6$$

For  $X = 60$ ,

$$Y = 0.135(60) + 0.6 = 8.1 + 0.6 = 8.7$$

$\therefore$  The advertisement expenditure should be ₹ 8.7 crores if the firm proposes a sales target ₹ 60 crores

### Exercise 3.2 | Q 12 | Page 48

For certain bivariate data the following information is available.

	<b>X</b>	<b>Y</b>
<b>Mean</b>	13	17
<b>S.D.</b>	3	2

Correlation coefficient between x and y is 0.6. estimate x when y = 15 and estimate y when x = 10.

**Solution:**

Given,  $\bar{x} = 13, \bar{y} = 1, \sigma_X = 3, \sigma_Y = 2, r = 0.6$

$$b_{YX} = r \frac{\sigma_Y}{\sigma_X} = 0.6 \times \frac{2}{3} = 0.4$$

$$b_{XY} = r \frac{\sigma_X}{\sigma_Y} = 0.6 \times \frac{3}{2} = 0.9$$

The regression equation of X on Y is given by

$$(X - \bar{x}) = b_{XY}(Y - \bar{y})$$

$$(X - 13) = 0.9(Y - 17)$$

$$X - 13 = 0.9Y - 15.3$$

$$X = 0.9Y - 15.3 + 13$$

$$X = -2.3 + 0.9Y \quad \dots(i)$$

For  $Y = 15$ , from equation (i) we get

$$X = -2.3 + (0.9)(15) = -2.3 + 13.5 = 11.2$$

The regression equation of Y on X is given by

$$(Y - \bar{y}) = b_{YX}(X - \bar{x})$$

$$(Y - 17) = 0.4(X - 13)$$

$$Y - 17 = 0.4X - 5.2$$

$$Y = 0.4X - 5.2 + 17$$

$$Y = 11.8 + 0.4X \quad \dots(ii)$$

For  $X = 10$ , from equation (ii) we get

$$Y = 11.8 + 0.4(10) = 11.8 + 4 = 15.8$$

### EXERCISE 3.3 [PAGES 49 - 50]

#### Exercise 3.3 | Q 1 | Page 49

From the two regression equations, find  $r$ ,  $\bar{x}$  and  $\bar{y}$ .  $4y = 9x + 15$   
and  $25x = 4y + 17$

**Solution:** Given regression equations are

$$4y = 9x + 15$$

$$\text{i.e., } -9x + 4y = 15 \quad \dots(i)$$

$$\text{and } 25x = 4y + 17$$

$$\text{i.e., } 25x - 4y = 17 \quad \dots(ii)$$

Adding equations (i) and (ii), we get

$$-9x + 4y = 15$$

$$\underline{25x - 4y = 17}$$

$$16x = 32$$

$$\therefore x = 2$$

Substituting  $x = 2$  in (i), we get

$$-9(2) + 4y = 15$$

$$\therefore -18 + 4y = 15$$

$$\therefore 4y = 33$$

$$\therefore y = 8.25$$

Since the point of intersection of two regression lines is



$$(\bar{x}, \bar{y}), \bar{x} = 2 \text{ and } \bar{y} = 8.25$$

Let  $4y = 9x + 15$  be the regression equation of Y on X.

$$\therefore \text{The equation becomes } Y = \frac{9}{2}X + \frac{15}{4}$$

Comparing it with  $Y = b_{YX}X + a$ , we get

$$b_{YX} = \frac{9}{2} = 2.25$$

Now, the other equation, i.e.,  $25x = 4y + 17$  is the regression equation of X on Y.

$$\therefore \text{The equation becomes } X = \frac{4}{25}Y + \frac{17}{25}$$

Comparing it with  $X = b_{XY}Y + a'$ , we get

$$b_{XY} = \frac{4}{25} = 0.16$$

$$\begin{aligned} r &= \pm \sqrt{b_{XY} \cdot b_{YX}} \\ &= \pm \sqrt{0.16 \times 2.25} \\ &= \pm \sqrt{0.36} = \pm 0.6 \end{aligned}$$

Since  $b_{YX}$  and  $b_{XY}$  are positive,

$r$  is also positive.

$$\therefore r = 0.6$$

$$\therefore \bar{x} = 2 \text{ and } \bar{y} = 8.25 \text{ and } r = 0.6$$

**Exercise 3.3 | Q 2 | Page 49**

In a partially destroyed laboratory record of an analysis of regression data, the following data are legible:

Variance of  $X = 9$

Regression equations:

$$8x - 10y + 66 = 0$$

$$\text{and } 40x - 18y = 214.$$

Find on the basis of above information

- i. The mean values of  $X$  and  $Y$ .
- ii. Correlation coefficient between  $X$  and  $Y$ .
- iii. Standard deviation of  $Y$ .

**Solution:**

$$\text{Given, } \sigma_X^2 = 9$$

$$\therefore \sigma_X = 3$$

**(i)** The two regression equations are

$$8x - 10y + 66 = 0$$

$$\text{i.e., } 8x - 10y = -66 \quad \dots(i)$$

$$\text{and } 40x - 18y = 214 \quad \dots(ii)$$

By  $5 \times (i) - (ii)$ , we get

$$40x - 50y = -330$$

$$40x - 18y = 214$$

$$(-) \underline{\quad} (+) \underline{\quad} (-) \underline{\quad}$$

$$-32y = -544$$

$$\therefore y = \frac{544}{32} = 17$$

Substituting  $y = 17$  in (i), we get

$$8x - 10 \times 17 = -66$$

$$\therefore 8x - 170 = -66$$

$$\therefore 8x = -66 + 170$$

$$\therefore 8x = 104$$

$$\therefore x = \frac{104}{8} = 13$$

Since the point of intersection of two regression lines is  $(\bar{x}, \bar{y})$ ,

$\bar{x}$  = mean value of  $X = 13$ , and

$\bar{y}$  = mean value of  $Y = 17$ .

**(ii)** Let  $8x - 10y + 66 = 0$  be the regression equation of  $Y$  on  $X$ .

$\therefore$  The equation becomes  $10Y = 8X + 66$

$$\text{i.e., } Y = \frac{8}{10}X + \frac{66}{10}$$

$$\text{i.e., } Y = \frac{4}{5}X + \frac{33}{5}$$

Comparing it with  $Y = b_{YX}X + a$ , we get

$$b_{YX} = \frac{4}{5}$$

Now, the other equation, i.e.,  $40x - 18y = 214$  is the regression equation of  $X$  on  $Y$ .

$$\therefore \text{The equation becomes } X = \frac{18}{40}Y + \frac{214}{40}$$

$$\text{i.e., } X = \frac{9}{20}Y + \frac{107}{20}$$

Comparing it with  $X = b_{XY}Y + a'$ , we get

$$b_{XY} = \frac{9}{20}$$

$$r = \pm \sqrt{b_{XY} \cdot b_{YX}}$$

$$\therefore r = \pm \sqrt{\frac{9}{20} \times \frac{4}{5}} = \pm \sqrt{\frac{9}{25}} = \pm \frac{3}{5} = \pm 0.6$$

Since  $b_{YX}$  and  $b_{XY}$  both are positive,

$r$  is also positive.

$$\therefore r = 0.6$$

$$\text{(iii) } b_{YX} = r \frac{\sigma_Y}{\sigma_X}$$

$$\therefore \frac{4}{5} = 0.6 \times \frac{\sigma_Y}{3}$$

$$\therefore \frac{4}{5} = \frac{\sigma_Y}{5}$$

$$\therefore \sigma_Y = 4$$

### Exercise 3.3 | Q 3 | Page 50

For 50 students of a class, the regression equation of marks in statistics ( $X$ ) on the marks in Accountancy ( $Y$ ) is  $3y - 5x + 180 = 0$ .

The mean marks in accountancy is 44 and the variance of marks in statistics is  $(9/16)^{\text{th}}$  of the variance of marks in accountancy. Find the mean marks in statistics and the correlation coefficient between marks in two subjects.

**Solution:** Given,  $n = 50$ ,  $X$  = marks in Statistics,

Y = marks in Accountancy,

Regression equation of X on Y is

$$3y - 5x + 180 = 0,$$

$$\bar{y} = 44, \sigma_X^2 = \frac{9}{16}\sigma_Y^2$$

Now,  $3y - 5x + 180 = 0$  is the regression equation of X on Y.

$\therefore$  The equation becomes  $5X = 3Y + 180$

$$\text{i.e., } X = \frac{3}{5}Y + \frac{180}{5}$$

Comparing it with  $X = b_{XY} Y + a'$ , we get

$$b_{XY} = \frac{3}{5}, a' = \frac{180}{5} = 36$$

$$a' = \bar{x} - b_{XY} \bar{y}$$

$$\therefore 36 = \bar{x} - \frac{3}{5} \times 44$$

$$\therefore 36 = \bar{x} - 26.4$$

$$\therefore \bar{x} = 36 + 26.4 = 62.4$$

$$\text{Also, } \sigma_X^2 = \frac{9}{16}\sigma_Y^2$$

$$\therefore \frac{\sigma_X^2}{\sigma_Y^2} = \frac{9}{16}$$

$$\therefore \frac{\sigma_X}{\sigma_Y} = \frac{3}{4}$$

$$b_{XY} = r \times \frac{\sigma_X}{\sigma_Y}$$

$$\therefore \frac{3}{5} = r \times \frac{3}{4}$$

$$\therefore \frac{3}{5} \times \frac{4}{3} = r$$

$$\therefore r = \frac{4}{5} = 0.8$$

$\therefore$  Mean marks in statistics ( $\bar{x}$ ) are 62.4 and correlation coefficient (r) between marks in the two subjects is 0.8.

### Exercise 3.3 | Q 4 | Page 50

For bivariate data, the regression coefficient of Y on X is 0.4 and the regression coefficient of X on Y is 0.9. Find the value of the variance of Y if the variance of X is 9.

**Solution:**

$$\text{Given, } b_{YX} = 0.4, b_{XY} = 0.9, \sigma_X^2 = 9$$

$$\therefore \sigma_X = 3$$

$$r = \pm \sqrt{b_{XY} \cdot b_{YX}}$$

$$= \pm \sqrt{0.9 \times 0.4} = \pm \sqrt{0.36} = \pm 0.6$$

Since  $b_{YX}$  and  $b_{XY}$  both are positive,

r is also positive.

$$\therefore r = 0.6$$

$$\text{Now, } b_{YX} = r \times \frac{\sigma_Y}{\sigma_X}$$

$$\therefore 0.4 = 0.6 \times \frac{\sigma_Y}{3}$$

$$\therefore 0.4 = 0.2 \times \sigma_Y$$

$$\therefore \sigma_Y = \frac{0.4}{0.2} = 2$$

$$\therefore \sigma_Y^2 = 4$$

$\therefore$  The value of variance of Y is 4.

### Exercise 3.3 | Q 5 | Page 50

The equations of two regression lines are

$$2x + 3y - 6 = 0$$

and  $2x + 2y - 12 = 0$  Find

i. Correlation coefficient

ii.  $\frac{\sigma_X}{\sigma_Y}$

**Solution:** The given regression equations are

$$2x + 3y - 6 = 0 \text{ and } 2x + 2y - 12 = 0$$

(i) Let  $2x + 3y - 6 = 0$  be the regression equation of Y on X

$\therefore$  The equation becomes  $3Y = -2X + 6$

$$\text{i.e., } Y = \frac{-2}{3}X + \frac{6}{3}$$

Comparing it with  $Y = b_{YX}X + a$ , we get

$$b_{YX} = -\frac{2}{3}$$

Now, the other equation, i.e.,  $2x + 2y - 12 = 0$  is the regression equation of X on Y.

∴ The equation becomes  $2X = -2Y + 12$

$$\text{i.e., } X = -\frac{2}{2}Y + \frac{12}{2}$$

Comparing it with  $X = b_{XY} Y + a'$  we get

$$b_{XY} = -\frac{2}{2} = -1$$

$$\therefore r = \pm \sqrt{b_{XY} \cdot b_{YX}}$$

$$= \pm \sqrt{-1 \cdot \left(-\frac{2}{3}\right)} = \pm \sqrt{\frac{2}{3}} = \pm 0.82$$

since  $b_{XY}$  and  $b_{YX}$  are negative,

$r$  is also negative.

$$\therefore r = -0.82$$

$$\text{(ii) } b_{XY} = r \frac{\sigma_X}{\sigma_Y}$$

$$\therefore -1 = -0.82 \times \frac{\sigma_X}{\sigma_Y}$$

$$\therefore \frac{\sigma_X}{\sigma_Y} = \frac{-1}{-0.82}$$

$$\therefore \frac{\sigma_X}{\sigma_Y} = 1.22$$

### Exercise 3.3 | Q 6 | Page 50

For a bivariate data:  $\bar{x} = 53$ ,  $\bar{y} = 28$ ,  $b_{YX} = -1.5$  and  $b_{XY} = -0.2$ .

Estimate  $Y$  when  $X = 50$ .

**Solution:**



Given,  $\bar{x} = 53$ ,  $\bar{y} = 28$ ,  $b_{YX} = -1.5$  and  $b_{XY} = -0.2$ .

$$a = \bar{y} - b_{YX} \bar{x}$$

$$\therefore a = 28 - (-1.5)(53) = 28 + 79.5 = 107.5$$

Now, the regression equation Y on X is

$$Y = a + b_{YX} X$$

$$\text{i.e., } Y = 107.5 + (-1.5)X$$

$$\text{i.e., } Y = 107.5 - 1.5 X$$

When  $X = 50$ , we get

$$Y = 107.5 - 1.5 \times 50 = 107.5 - 75 = 32.5$$

### Exercise 3.3 | Q 7 | Page 50

The equations of two regression lines are  $x - 4y = 5$  and  $16y - x = 64$ . Find means of X and Y. Also, find correlation coefficient between X and Y.

**Solution:** Given equations of regression lines are

$$x - 4y = 5 \quad \dots(i)$$

$$16y - x = 64$$

$$\text{i.e., } -x + 16y = 64 \quad \dots(ii)$$

Adding (i) and (ii), we get

$$x - 4y = 5$$

$$\underline{-x + 16y = 64}$$

$$12y = 69$$

$$\therefore y = \frac{69}{12} = 5.75$$

Substituting  $y = 5.75$  in (i), we get

$$x - 4(5.75) = 5$$

$$\therefore x - 23 = 5$$

$$\therefore x = 5 + 23 = 28$$

Since the point of intersection of two regression lines is  $(\bar{x}, \bar{y})$ ,

$$\therefore \bar{x} = 28 \text{ and } \bar{y} = 5.75$$

Let,  $x - 4y = 5$  be the regression equation of X on Y

$$\therefore \text{The equation becomes } X = 4Y + 5$$

Comparing it with  $X = b_{XY} Y + a'$ , we get

$$b_{XY} = 4$$

Now, the other equation i.e.  $16y - x = 64$  is regression equation of Y on X

$$\therefore \text{The equation becomes } 16Y = X + 64$$

$$\text{i.e., } Y = \frac{1}{16}X + \frac{64}{16}$$

Comparing it with  $Y = b_{YX} X + a$ , we get

$$b_{YX} = \frac{1}{16}$$

$$r = \pm \sqrt{b_{XY} \cdot b_{YX}}$$

$$= \pm \sqrt{4 \times \frac{1}{16}} = \pm \sqrt{\frac{1}{4}} = \pm \frac{1}{2} = \pm 0.5$$

Since  $b_{XY}$  and  $b_{YX}$  both are positive,

$r$  is also positive.

$$\therefore r = 0.5$$

### Exercise 3.3 | Q 8 | Page 50

In a partially destroyed record, the following data are available: variance of  $X = 25$ , Regression equation of  $Y$  on  $X$  is  $5y - x = 22$  and regression equation of  $X$  on  $Y$  is  $64x - 45y = 22$  Find

- Mean values of  $X$  and  $Y$
- Standard deviation of  $Y$
- Coefficient of correlation between  $X$  and  $Y$ .

**Solution:**

$$\text{Given, } \sigma_X^2 = 25$$

$$\therefore \sigma_X = 5$$

Regression equation of  $Y$  on  $X$  is

$$5y - x = 22$$

Regression equation of  $X$  on  $Y$  is

$$64x - 45y = 22$$

**(i)** Consider, the two regression equation

$$-x + 5y = 22 \quad \dots(i)$$

$$64x - 45y = 22 \quad \dots(ii)$$

By (i)  $\times$  + (ii), we get

$$\begin{array}{r} -9x + 45y = 198 \\ + 64x - 45y = 22 \\ \hline 55x = 220 \end{array}$$

$$\therefore x = 4$$

Substituting  $x = 4$  in (i), we get

$$-4 + 5y = 22$$

$$\therefore 5y = 22 + 4$$

$$\therefore y = \frac{26}{5} = 5.2$$

Since the point of intersection of two regression lines is  $(\bar{x}, \bar{y})$ ,

$\bar{x}$  = mean value of  $X = 4$  and

$\bar{y}$  = mean value of  $Y = 5.2$

**(ii)** To find standard deviation of  $Y$  we should first find the coefficient of correlation between  $X$  and  $Y$ .

Regression equation of  $Y$  on  $X$  is

$$5y - x = 22$$

$$\text{i.e., } 5Y = X + 22$$

$$\text{i.e., } Y = \frac{X}{5} + \frac{22}{5}$$

Comparing it with  $Y = b_{YX} X + a$ , we get

$$b_{YX} = 1/5$$

Now, regression equation of  $X$  on  $Y$  is

$$64x - 45y = 22$$

$$\text{i.e., } 64X - 45Y = 22$$

$$\text{i.e., } 64X = 45Y + 22$$

$$\text{i.e., } X = \frac{45Y}{64} + \frac{22}{64}$$

Comparing it with  $X = b_{XY} Y + a'$ , we get

$$b_{XY} = \frac{45}{64}$$

$$r = \pm \sqrt{b_{XY} \cdot b_{YX}}$$

$$= \pm \sqrt{\left(\frac{1}{5}\right) \left(\frac{45}{64}\right)} = \pm \sqrt{\frac{9}{64}} = \pm \frac{3}{8}$$

Since  $b_{YX}$  and  $b_{XY}$  are positive,

$r$  is also positive.

$$\therefore r = \frac{3}{8}$$

$$\text{Now, } b_{YX} = r \frac{\sigma_Y}{\sigma_X}$$

$$\therefore \frac{1}{5} = \frac{3}{8} \times \frac{\sigma_Y}{5}$$

$$\therefore \sigma_Y = \frac{1}{5} \times \frac{8}{3} \times 5$$

$$\therefore \sigma_Y = \text{Standard deviation of } Y = \frac{8}{3}$$

**(iii)** The correlation coefficient of  $X$  and  $Y$  is

$$r = \frac{8}{3} = 0.375$$

**Exercise 3.3 | Q 9 | Page 50**

If the two regression lines for a bivariate data are  $2x = y + 15$  (x on y) and  $4y = 3x + 25$  (y on x), find

i.  $\bar{x}$ ,

ii.  $\bar{y}$ ,

iii.  $b_{YX}$

iv.  $b_{XY}$

v.  $r$  [Given  $\sqrt{0.375} = 0.61$ ]

**Solution:** Given,

regression equation of X on Y is  $2x = y + 15$ ,

i.e.,  $2x - y = 15$  ... (i)

regression equation of Y on X is  $4y = 3x + 25$ ,

i.e.,  $-3x + 4y = 25$  ... (ii)

By (i)  $\times$  4 + (ii) we get

$$\begin{array}{r} 8x - 4y = 60 \\ + -3x + 4y = 25 \\ \hline 5x = 85 \end{array}$$

$\therefore x = 17$

Substituting the value of  $x = 17$  in (i), we get

$2(17) - y = 15$

$\therefore 34 - y = 15$

$\therefore y = 34 - 15 = 19$

Since the point of intersection of two regression lines is  $(\bar{x}, \bar{y})$ ,

**(i)**  $\bar{x} = 17$

**(ii)**  $\bar{y} = 19$

**(iii)** Regression equation of Y on X is  $4y = 3x + 25$

i.e.,  $4Y = 3X + 25$

i.e.,  $Y = \frac{3}{4}X + \frac{25}{4}$

Comparing it with  $Y = b_{YX}X + a$ , we get

$$b_{YX} = \frac{3}{4}$$

**(iv)** Regression equation of X on Y is  $2x = y + 15$

i.e.,  $2X = Y + 15$

i.e.,  $X = \frac{Y}{2} + \frac{15}{2}$

Comparing it with  $X = b_{XY}Y + a'$  we get,

$$b_{XY} = \frac{1}{2}$$

**(v)**  $r = \pm\sqrt{b_{XY} \cdot b_{YX}}$

$$= \pm\sqrt{\left(\frac{1}{2}\right) \cdot \left(\frac{3}{4}\right)} = \pm\sqrt{0.375} = \pm 0.61$$

Since  $b_{YX}$  and  $b_{XY}$  both are positive,

$r$  is also positive.

$$\therefore r = 0.61$$

### Exercise 3.3 | Q 10 | Page 50

The two regression equations are  $5x - 6y + 90 = 0$  and  $15x - 8y - 130 = 0$ . Find  $\bar{x}$ ,  $\bar{y}$ ,  $r$ .

**Solution:**

Given, the two regression equations are

$$5x - 6y + 90 = 0$$

$$\text{i.e., } 5x - 6y = -90 \quad \dots(i)$$

$$\text{and } 15x - 8y - 130 = 0$$

$$\text{i.e., } 15x - 8y = 130 \quad \dots(ii)$$

By (i)  $\times 3 -$  (ii), we get

$$15x - 18y = -270$$

$$15x - 8y = 130$$

$$\begin{array}{r} - \quad + \quad - \\ -10y = -400 \end{array}$$

$$\therefore y = 40$$

Substituting  $y = 40$  in (i), we get

$$5x - 6(40) = -90$$

$$\therefore 5x - 240 = -90$$

$$\therefore 5x = -90 + 240 = 150$$

$$\therefore x = 30$$

Since the point of intersection of two regression lines is  $(\bar{x}, \bar{y})$ ,

$$\therefore \bar{x} = 30 \text{ and } \bar{y} = 40$$

Now, let  $5x - 6y + 90 = 0$  be the regression equation of Y on X.

$$\therefore \text{The equation becomes } 6Y = 5X + 90$$



$$\text{i.e., } Y = \frac{5}{6}X + \frac{90}{6}$$

Comparing it with  $Y = b_{YX}X + a$ , we get

$$\therefore b_{YX} = \frac{5}{6}$$

Now, other equation  $15x - 8y - 130 = 0$  be the regression equation of X on Y.

$\therefore$  The equation becomes  $15X = 8Y + 130$

$$\text{i.e., } X = \frac{8}{15}Y + \frac{130}{15}$$

Comparing it with  $X = b_{XY}Y + a'$ , we get

$$\therefore b_{XY} = \frac{8}{15}$$

$$\therefore r = \pm \sqrt{b_{XY} \cdot b_{YX}}$$

$$= \pm \sqrt{\frac{8}{15} \cdot \frac{5}{6}} = \pm \sqrt{\frac{4}{9}} = \pm \frac{2}{3}$$

Since  $b_{YX}$  and  $b_{XY}$  both are positive,

$r$  is positive.

$$\therefore r = \frac{2}{3}$$

### Exercise 3.3 | Q 11 | Page 50

Two lines of regression are  $10x + 3y - 62 = 0$  and  $6x + 5y - 50 = 0$ . Identify the regression of  $x$  on  $y$ . Hence find  $\bar{x}$ ,  $\bar{y}$  and  $r$ .

**Solution:**

Given, two lines of regression are

$$10x + 3y - 62 = 0$$

$$\text{i.e., } 10x + 3y = 62 \quad \dots(i)$$

$$\text{and } 6x + 5y - 50 = 0$$

$$\text{i.e., } 6x + 5y = 50 \quad \dots(ii)$$

By (i)  $\times 5$  - (ii)  $\times 3$ , we get

$$50x + 15y = 310$$

$$18x + 15y = 150$$

$$\begin{array}{r} - \quad - \quad - \\ 32x \quad = 160 \end{array}$$

$$\therefore x = 5$$

Substituting  $x = 5$  in (i) we get,

$$10(5) + 3y = 62$$

$$\therefore 50 + 3y = 62$$

$$\therefore 3y = 62 - 50 = 12$$

$$\therefore y = 4$$

Since the point of intersection of two regression lines is  $(\bar{x}, \bar{y})$ ,

$$\bar{x} = 5 \quad \text{and} \quad \bar{y} = 4$$

Now,

Let  $10x + 3y - 62 = 0$  be the regression equation of X on Y.

$\therefore$  The equation becomes  $10x = -3y + 62$

$$\text{i.e., } 10X = -3Y + 62$$

$$\text{i.e., } X = -\frac{3}{10}Y + \frac{62}{10}$$

Comparing it with  $X = b_{XY} Y + a$ , we get

$$\therefore b_{XY} = -\frac{3}{10}$$

Now, other equation  $6x + 5y - 50 = 0$  be the regression equation of Y on X.

$\therefore$  The equation becomes  $5y = -6x + 50$

$$\text{i.e., } 5Y = -6X + 50$$

$$\text{i.e., } Y = -\frac{6}{5}x + \frac{50}{5}$$

Comparing it with  $Y = b_{YX} X + a'$ , we get

$$b_{YX} = -\frac{6}{5}$$

$$\text{Now, } b_{YX} \cdot b_{XY} = \left(-\frac{3}{10}\right)\left(-\frac{6}{5}\right) = \frac{9}{25}$$

$$\text{i.e., } b_{XY} \cdot b_{YX} < 1$$

$\therefore$  Assumption of regression equations is true.

$$\therefore r = \pm \sqrt{b_{XY} \cdot b_{YX}} = \pm \sqrt{\frac{9}{25}} = \pm \frac{3}{5}$$

Since  $b_{YX}$  and  $b_{XY}$  both are negative,

$r$  is negative.

$$\therefore r = -\frac{3}{5} = -0.6$$

### Exercise 3.3 | Q 12 | Page 50

For certain X and Y series, which are correlated the two lines of regression are  $10y = 3x + 170$  and  $5x + 70 = 6y$ . Find the correlation coefficient between them. Find the mean values of X and Y.

**Solution:** Let  $10y = 3x + 170$  be the regression equation of Y on X.

$\therefore$  The equation becomes  $10y = 3x + 170$

$$\text{i.e., } Y = \frac{3}{10}X + \frac{170}{10}$$

Comparing it with  $Y = b_{YX}X + a$ , we get

$$b_{YX} = \frac{3}{10}$$

Now other equation  $5x + 70 = 6y$  be the regression equation of X on Y.

$\therefore$  The equation becomes  $5x = 6y - 70$

$$\text{i.e., } X = \frac{6}{5}Y - \frac{70}{5}$$

Comparing it with  $X = b_{XY}Y + a'$ , we get

$$b_{XY} = \frac{6}{5}$$

$$\therefore r = \pm \sqrt{b_{XY} \cdot b_{YX}} = \pm \sqrt{\frac{6}{5} \times \frac{3}{10}} = \pm \sqrt{\frac{9}{25}} = \pm \frac{3}{5}$$

Since  $b_{YX}$  and  $b_{XY}$  both are positive,  
r is positive.

$$\therefore r = \frac{3}{5} = 0.6$$

Now, two correlated lines of regression are

$$10y = 3x + 170$$

$$\text{i.e., } -3x + 10y = 170 \quad \dots(i)$$

$$\text{and } 5x + 70 = 6y$$

$$\text{i.e., } 5x - 6y = -70 \quad \dots(ii)$$

By (i)  $\times 5$  + (ii)  $\times 3$ , we get

$$-15x + 50y = 850$$

$$+ \underline{15x - 18y = -210}$$

$$32y = 640$$

$$\therefore y = 20$$

Substituting  $y = 20$  in equation (i), we get

$$-3x + 10(20) = 170$$

$$\therefore -3x + 200 = 170$$

$$\therefore 3x = 200 - 170$$

$$\therefore x = 10$$

Since the point of intersection of two regression lines is  $(\bar{x}, \bar{y})$ ,

$\bar{x}$  = mean value of  $X = 10$ , and

$\bar{y}$  = mean value of  $Y = 20$ .

**Exercise 3.3 | Q 13 | Page 50**

Regression equations of two series are  $2x - y - 15 = 0$  and  $3x - 4y + 25 = 0$ . Find  $\bar{x}$ ,  $\bar{y}$  and regression coefficients. Also find coefficients of correlation. [Given  $\sqrt{0.375} = 0.61$ ]

**Solution:** Given regression equation of two series are

$$2x - y - 15 = 0$$

$$\text{i.e., } 2x - y = 15 \quad \dots(i)$$

$$\text{and } 3x - 4y + 25 = 0$$

$$\text{i.e., } 3x - 4y = -25 \quad \dots(ii)$$

By (i)  $\times$  - (ii), we get

$$8x - 4y = 60$$

$$3x - 4y = -25$$

$$\begin{array}{r} - \quad + \quad + \\ \hline 5x = 85 \end{array}$$

$$\therefore x = 17$$

Substituting  $x = 17$  in in equation (i), we have

$$2(17) - y = 15$$

$$\therefore 34 - y = 15$$

$$\therefore y = 34 - 15 = 19$$

Since the point of intersection of two regression lines is  $(\bar{x}, \bar{y})$ ,

$$\bar{x} = 17 \quad \text{and} \quad \bar{y} = 19$$

Now,

Let  $2x - y - 15 = 0$  be the regression equation of X on Y.

$$\therefore \text{The equation becomes } 2x = y + 15$$

$$\text{i.e., } X = \frac{Y}{2} + \frac{15}{2}$$

Comparing it with  $X = b_{XY} Y + a$ , we get

$$\therefore b_{XY} = \frac{1}{2}$$

Now, other equation  $3x - 4y + 25 = 0$  be the regression equation of Y on X.

$\therefore$  The equation becomes  $4y = 3x + 25$

$$\text{i.e., } Y = \frac{3}{4}X + \frac{25}{4}$$

Comparing it with  $Y = b_{YX} X + a'$ , we get

$$\therefore b_{YX} = \frac{3}{4}$$

$$\therefore r = \pm \sqrt{b_{XY} \cdot b_{YX}}$$

$$= \pm \sqrt{\frac{1}{2} \cdot \frac{3}{4}} = \pm \sqrt{0.375} = \pm 0.61$$

Since  $b_{YX}$  and  $b_{XY}$  both are positive,

r is positive.

$$\therefore r = 0.61$$

### Exercise 3.3 | Q 14 | Page 50

The two regression lines between height (X) in inches and weight (Y) in kgs of girls are,

$$4y - 15x + 500 = 0$$

$$\text{and } 20x - 3y - 900 = 0$$

Find the mean height and weight of the group. Also, estimate the weight of a girl whose height is 70 inches.

**Solution:** Given,  $X$  = Height (in inches),  $Y$  = weight (in Kg)

The equation of regression are

$$4y - 15x + 500 = 0$$

$$\text{i.e., } -15x + 4y = -500 \quad \dots(i)$$

$$\text{and } 20x - 3y - 900 = 0$$

$$\text{i.e., } 20x - 3y = 900 \quad \dots(ii)$$

By  $3 \times (i) + 4 \times (ii)$ , we get

$$-45x + 12y = -1500$$

$$+ 80x - 12y = 3600$$

$$\hline 35x = 2100$$

$$\therefore x = 60$$

Substituting  $x = 60$  in (i), we get

$$-15(60) + 4y = -500$$

$$\therefore 4y = 900 - 500$$

$$\therefore y = 100$$

Since the point of intersection of two regression lines is  $\bar{x}, \bar{y}$ ,

$\bar{x}$  = mean height of the group = 60 inches, and

$\bar{y}$  = mean weight of the group = 100 kg.

Let  $4y - 15x + 500 = 0$  be the regression equation of  $Y$  on  $X$ .

$\therefore$  The equation becomes  $4y = 15x - 500$

$$\text{i.e., } Y = \frac{15}{4}X - \frac{500}{4} \quad \dots(i)$$



Comparing it with  $Y = b_{YX} X + a$ , we get

$$\therefore b_{YX} = \frac{15}{4}$$

$\therefore$  Now, other equation  $20x - 3y - 900 = 0$  be the regression equation of X on Y

$\therefore$  The equation becomes  $20x - 3y - 900 = 0$

$$\text{i.e., } 20x = 3y + 900$$

$$X = \frac{3}{20}Y + \frac{900}{20}$$

Comparing it with  $X = b_{XY} Y + a'$ ,

$$\therefore b_{XY} = \frac{3}{20}$$

$$\text{Now, } b_{YX} \cdot b_{XY} = \frac{15}{4} \cdot \frac{3}{20} = 0.5625$$

$$\text{i.e., } b_{XY} \cdot b_{YX} < 1$$

$\therefore$  Assumption of regression equations is true.

Now, substituting  $x = 70$  in (i) we get

$$y = \frac{15}{4} \times 70 - \frac{500}{4} = \frac{1050 - 500}{4} = \frac{550}{4} = 137.5$$

$\therefore$  Weight of girl having height 70 inches is 137.5 kg

### MISCELLANEOUS EXERCISE 3 [PAGES 51 - 54]

#### Miscellaneous Exercise 3 | Q 1.01 | Page 51

Choose the correct alternative.

Regression analysis is the theory of

1. Estimation
2. Prediction
3. **Estimation and Prediction**
4. Calculation

**Solution:** Regression analysis is the theory of Estimation and Prediction.

**Miscellaneous Exercise 3 | Q 1.02 | Page 51**

**Choose the correct alternative.**

We can estimate the value of one variable with the help of other known variable only if they are

1. **Correlated**
2. Positively correlated
3. Negatively correlated
4. Uncorrelated

**Solution:** We can estimate the value of one variable with the help of other known variable only if they are correlated.

**Miscellaneous Exercise 3 | Q 1.03 | Page 51**

**Choose the correct alternative.**

There are \_\_\_\_\_ types of regression equations.

1. 4
2. **2**
3. 3
4. 1

**Solution:** There are 2 types of regression equations.

**Miscellaneous Exercise 3 | Q 1.04 | Page 51**

**Choose the correct alternative.**

In the regression equation of Y on X

1. **X is independent and Y is dependent.**
2. Y is independent and X is dependent.
3. Both X and Y are independent.
4. Both X and Y are dependent.



**Solution:** In the regression equation of Y on X, X is independent and Y is dependent.

**Miscellaneous Exercise 3 | Q 1.05 | Page 52**

**Choose the correct alternative.**

In the regression equation of X on Y

1. X is independent and Y is dependent.
2. **Y is independent and X is dependent.**
3. Both X and Y are independent.
4. Both X and Y are dependent.

**Solution:** Y is independent and X is dependent.

**Miscellaneous Exercise 3 | Q 1.06 | Page 52**

**Choose the correct alternative.**

$b_{XY}$  is \_\_\_\_\_

1. Regression coefficient of Y on X
2. **Regression coefficient of X on Y**
3. Correlation coefficient between X and Y
4. Covariance between X and Y

**Solution:**  $b_{XY}$  is regression coefficient of X on Y.

**Miscellaneous Exercise 3 | Q 1.07 | Page 52**

**Choose the correct alternative.**

$b_{YX}$  is \_\_\_\_\_.

1. **Regression coefficient of Y on X**
2. Regression coefficient of X on Y
3. Correlation coefficient between X and Y
4. Covariance between X and Y

**Solution:**  $b_{YX}$  is regression coefficient of Y on X.

**Miscellaneous Exercise 3 | Q 1.08 | Page 52**

**Choose the correct alternative.**

'r' is \_\_\_\_\_.

1. Regression coefficient of Y on X
2. Regression coefficient of X on Y
3. **Correlation coefficient between X and Y**
4. Covariance between X and Y

**Solution:** 'r' is correlation coefficient between X and Y.

### Miscellaneous Exercise 3 | Q 1.09 | Page 52

**Choose the correct alternative.**

$b_{XY} \cdot b_{YX}$  is \_\_\_\_\_.

1.  $v(x)$
2.  $\sigma_x$
3.  **$r^2$**
4.  $(\sigma_y)^2$

**Solution:**  $b_{XY} \cdot b_{YX}$  is  $r^2$

### Miscellaneous Exercise 3 | Q 1.1 | Page 52

**Choose the correct alternative.**

$b_{YX} > 1$  then  $b_{XY}$  is \_\_\_\_\_

1.  $> 1$
2.  **$< 1$**
3.  $> 0$
4.  $< 0$

**Solution:**  $b_{YX} > 1$  then  $b_{XY}$  is  $\leq 1$ .

### Miscellaneous Exercise 3 | Q 1.11 | Page 52

**Choose the correct alternative.**

$|b_{xy} + b_{yx}| \geq$  \_\_\_\_\_

1.  $|r|$
2.  **$2|r|$**
3.  $r$
4.  $2r$

**Solution:**  $|b_{xy} + b_{yx}| \geq$   $2|r|$ .

### Miscellaneous Exercise 3 | Q 1.12 | Page 52

Choose the correct alternative.

$b_{xy}$  and  $b_{yx}$  are \_\_\_\_\_

1. Independent of change of origin and scale
2. **Independent of change of origin but not of scale**
3. Independent of change of scale but not of origin
4. Affected by change of origin and scale

**Solution:**  $b_{xy}$  and  $b_{yx}$  are independent of change of origin but not of scale.

### Miscellaneous Exercise 3 | Q 1.13 | Page 52

Choose the correct alternative.

If  $u = \frac{x - a}{c}$  and  $v = \frac{y - b}{d}$  then  $b_{yx} =$  \_\_\_\_\_

Options

$$\frac{d}{c} b_{vu}$$

$$\frac{c}{d} b_{vu}$$

$$\frac{a}{b} b_{vu}$$

$$\frac{b}{a} b_{vu}$$

**Solution:**

If  $u = \frac{x - a}{c}$  and  $v = \frac{y - b}{d}$  then  $b_{yx} = \underline{\frac{d}{c} b_{vu}}$

### Miscellaneous Exercise 3 | Q 1.14 | Page 52

**Choose the correct alternative.**

If  $u = \frac{x - a}{c}$  and  $v = \frac{y - b}{d}$  then  $b_{xy} = \underline{\hspace{2cm}}$

Options

$$\frac{d}{c} b_{uv}$$

$$\frac{c}{d} b_{uv}$$

$$\frac{a}{b} b_{uv}$$

$$\frac{b}{a} b_{uv}$$

**Solution:**

If  $u = \frac{x - a}{c}$  and  $v = \frac{y - b}{d}$  then  $b_{xy} = \underline{\frac{c}{d} b_{uv}}$

**Miscellaneous Exercise 3 | Q 1.15 | Page 52**

**Choose the correct alternative.**

$\text{Corr}(x, x) = \underline{\hspace{2cm}}$

1. 0
2. 1
3. -1
4. can't be found

**Solution:**  $\text{Corr}(x, x) = \underline{1}$ .

**Miscellaneous Exercise 3 | Q 1.16 | Page 52**

**Choose the correct alternative.**

$\text{Corr}(x, y) = \underline{\hspace{2cm}}$



1.  $\text{corr}(x, x)$
2.  $\text{corr}(y, y)$
3.  **$\text{corr}(y, x)$**
4.  $\text{cov}(y, x)$

**Solution:**  $\text{Corr}(x, y) = \underline{\text{corr}(y, x)}$ .

### Miscellaneous Exercise 3 | Q 1.17 | Page 52

**Choose the correct alternative.**

$$\text{Corr}\left(\frac{x - a}{c}, \frac{y - b}{d}\right) = -\text{corr}(x, y) \text{ if,}$$

1. **c and d are opposite in sign**
2. c and d are same in sign
3. a and b are opposite in sign
4. a and b are same in sign

**Solution:** c and d are opposite in sign

### Miscellaneous Exercise 3 | Q 1.18 | Page 52

**Choose the correct alternative.**

Regression equation of X on Y is \_\_\_\_\_

$$y - \bar{y} = b_{yx}(x - \bar{x})$$

$$\underline{x - \bar{x} = b_{xy}(y - \bar{y})}$$

$$y - \bar{y} = b_{xy}(x - \bar{x})$$

$$x - \bar{x} = b_{yx}(y - \bar{y})$$

**Solution:**

Regression equation of X on Y is  $x - \bar{x} = b_{xy}(y - \bar{y})$

### Miscellaneous Exercise 3 | Q 1.19 | Page 53

**Choose the correct alternative.**

Regression equation of Y on X is \_\_\_\_

Options

$$(y - \bar{y}) = b_{yx}(x - \bar{x})$$

$$(x - \bar{x}) = b_{xy}(y - \bar{y})$$

$$(y - \bar{y}) = b_{xy}(x - \bar{x})$$

$$(x - \bar{x}) = b_{yx}(y - \bar{y})$$

**Solution:**

Regression equation of Y on X is  $(y - \bar{y}) = b_{yx}(x - \bar{x})$

**Miscellaneous Exercise 3 | Q 1.2 | Page 53**

**Choose the correct alternative.**

$b_{yx} =$  \_\_\_\_\_

Options

$$r \frac{\sigma_x}{\sigma_y}$$

$$r \frac{\sigma_y}{\sigma_x}$$

$$\frac{1}{r} \frac{\sigma_y}{\sigma_x}$$

$$\frac{1}{r} \frac{\sigma_x}{\sigma_y}$$

**Solution:**

$$b_{yx} = r \frac{\sigma_y}{\sigma_x}$$



Miscellaneous Exercise 3 | Q 1.21 | Page 53

Choose the correct alternative.

$$b_{xy} = \underline{\hspace{2cm}}$$

Options

$$r \frac{\sigma_x}{\sigma_y}$$

$$r \frac{\sigma_y}{\sigma_x}$$

$$\frac{1}{r} \frac{\sigma_y}{\sigma_x}$$

$$\frac{1}{r} \frac{\sigma_x}{\sigma_y}$$

Solution:

$$b_{xy} = r \frac{\sigma_x}{\sigma_y}$$

Miscellaneous Exercise 3 | Q 1.22 | Page 53



**Choose the correct alternative.**

Cov (x, y) = \_\_\_\_\_

Options

$$\sum (x - \bar{x})(y - \bar{y})$$

$$\frac{\sum (x - \bar{x})(y - \bar{y})}{n}$$

$$\frac{\sum xy}{n} - \bar{x} \bar{y}$$

**both**  $\frac{\sum (x - \bar{x})(y - \bar{y})}{n}$  **and**  $\frac{\sum xy}{n} - \bar{x} \bar{y}$

**Solution:**

$$\text{Cov (x, y) = both } \frac{\sum (x - \bar{x})(y - \bar{y})}{n} \text{ and } \frac{\sum xy}{n} - \bar{x} \bar{y}$$

**Miscellaneous Exercise 3 | Q 1.23 | Page 53**

**Choose the correct alternative.**

If  $b_{xy} < 0$  and  $b_{yx} < 0$  then 'r' is \_\_\_\_\_

1.  $> 0$

2.  $< 0$

3.  $> 1$

4. not found

**Solution:** If  $b_{xy} < 0$  and  $b_{yx} < 0$  then 'r' is  $\leq 0$ .

**Miscellaneous Exercise 3 | Q 1.24 | Page 53**

**Choose the correct alternative.**

If equations of regression lines are  $3x + 2y - 26 = 0$  and  $6x + y - 31 = 0$  then means of x and y are \_\_\_\_\_

1. (7, 4)
2. (4, 7)
3. (2, 9)
4. (-4, 7)

**Solution:** If equations of regression lines are  $3x + 2y - 26 = 0$  and  $6x + y - 31 = 0$  then means of  $x$  and  $y$  are

**(4, 7).**

### Miscellaneous Exercise 3 | Q 2.01 | Page 53

**Fill in the blank:**

If  $b_{xy} < 0$  and  $b_{yx} < 0$  then 'r' is \_\_\_\_\_

**Solution:** If  $b_{xy} < 0$  and  $b_{yx} < 0$  then 'r' is **negative.**

### Miscellaneous Exercise 3 | Q 2.02 | Page 53

**Fill in the blank:**

Regression equation of Y on X is \_\_\_\_\_

**Solution:**

Regression equation of Y on X is  $(y - \bar{y}) = b_{YX}(x - \bar{x})$

### Miscellaneous Exercise 3 | Q 2.03 | Page 53

**Fill in the blank:**

Regression equation of X on Y is \_\_\_\_\_

**Solution:**

Regression equation of X on Y is  $(x - \bar{x}) = b_{XY}(y - \bar{y})$

### Miscellaneous Exercise 3 | Q 2.04 | Page 53

**Fill in the blank:**

There are \_\_\_\_\_ types of regression equations.

**Solution:** There are **2** types of regression equations.

### Miscellaneous Exercise 3 | Q 2.05 | Page 53

**Fill in the blank:**

$$\text{Corr}(x, -x) = \underline{\hspace{2cm}}$$

**Solution:**  $\text{Corr}(x, -x) = \underline{-1}$ .

**Miscellaneous Exercise 3 | Q 2.06 | Page 53**

**Fill in the blank:**

$$\text{If } u = \frac{x - a}{c} \text{ and } v = \frac{y - b}{d} \text{ then } b_{xy} = \underline{\hspace{2cm}}$$

**Solution:**

$$\text{If } u = \frac{x - a}{c} \text{ and } v = \frac{y - b}{d} \text{ then } b_{xy} = \underline{\frac{c}{d} b_{uv}}$$

**Miscellaneous Exercise 3 | Q 2.07 | Page 53**

**Fill in the blank:**

$$\text{If } u = \frac{x - a}{c} \text{ and } v = \frac{y - b}{d} \text{ then } b_{yx} = \underline{\hspace{2cm}}$$

**Solution:**

$$\text{If } u = \frac{x - a}{c} \text{ and } v = \frac{y - b}{d} \text{ then } b_{yx} = \underline{\frac{d}{c} b_{vu}}$$

**Miscellaneous Exercise 3 | Q 2.08 | Page 53**

**Fill in the blank:**

$$|b_{xy} + b_{yx}| \geq \underline{\hspace{2cm}}$$

**Solution:**  $|b_{xy} + b_{yx}| \geq \underline{2|r|}$

**Miscellaneous Exercise 3 | Q 2.09 | Page 53**

**Fill in the blank:**

$$\text{If } b_{yx} > 1 \text{ then } b_{xy} \text{ is } \underline{\hspace{2cm}}$$

**Solution:** If  $b_{yx} > 1$  then  $b_{xy}$  is  $\underline{\leq 1}$ .

### Miscellaneous Exercise 3 | Q 2.1 | Page 53

Fill in the blank:

$$b_{xy} \cdot b_{yx} = \underline{\hspace{2cm}}$$

**Solution:**  $b_{xy} \cdot b_{yx} = \underline{r^2}$

### Miscellaneous Exercise 3 | Q 3.01 | Page 53

State whether the following statement is True or False.

$$\text{Corr}(x, x) = 1$$

1. True
2. False

**Solution:** True.

### Miscellaneous Exercise 3 | Q 3.02 | Page 53

State whether the following statement is True or False.

$$\text{Regression equation of X on Y is } (y - \bar{y}) = b_{yx}(x - \bar{x})$$

1. True
2. False

**Solution:** False.

### Miscellaneous Exercise 3 | Q 3.03 | Page 53

State whether the following statement is True or False.

$$\text{Regression equation of Y on X is } (y - \bar{y}) = b_{yx}(x - \bar{x})$$

1. True
2. False

**Solution:** True.

### Miscellaneous Exercise 3 | Q 3.04 | Page 53

State whether the following statement is True or False.

$$\text{Corr}(x, y) = \text{Corr}(y, x)$$

1. True

2. False

**Solution:** True.

**Miscellaneous Exercise 3 | Q 3.05 | Page 53**

**State whether the following statement is True or False.**

$b_{xy}$  and  $b_{yx}$  are independent of change of origin and scale.

1. True
2. False

**Solution:** False.

**Miscellaneous Exercise 3 | Q 3.06 | Page 53**

**State whether the following statement is True or False.**

'r' is regression coefficient of Y on X

1. True
2. False

**Solution:** False.

**Miscellaneous Exercise 3 | Q 3.07 | Page 53**

**State whether the following statement is True or False.**

$b_{yx}$  is correlation coefficient between X and Y

1. True
2. False

**Solution:** False.

**Miscellaneous Exercise 3 | Q 3.08 | Page 53**

**State whether the following statement is True or False.**

If  $u = x - a$  and  $v = y - b$  then  $b_{xy} = b_{uv}$

1. True
2. False

**Solution:** True.

**Miscellaneous Exercise 3 | Q 3.09 | Page 53**

**State whether the following statement is True or False.**



If  $u = x - a$  and  $v = y - b$  then  $r_{xy} = r_{uv}$

1. True
2. False

**Solution:** True.

### Miscellaneous Exercise 3 | Q 3.1 | Page 53

**State whether the following statement is True or False.**

In the regression equation of Y on X,  $b_{yx}$  represents slope of the line.

1. True
2. False

**Solution:** True.

### Miscellaneous Exercise 3 | Q 4.01 | Page 54

The data obtained on X, the length of time in weeks that a promotional project has been in progress at a small business, and Y, the percentage increase in weekly sales over the period just prior to the beginning of the campaign.

X	1	2	3	4	1	3	1	2	3	4	2	4
Y	10	10	18	20	11	15	12	15	17	19	13	16

Find the equation of the regression line to predict the percentage increase in sales if the campaign has been in progress for 1.5 weeks.

**Solution:** Here, X = Length of time in weeks,  
Y = Percentage increase in weekly sales

X = $x_i$	Y = $y_i$	$x_i^2$	$x_i y_i$
1	10	1	10
2	10	4	20
3	18	9	54
4	20	16	80
1	11	1	11
3	15	9	45
1	12	1	12
2	15	4	30
3	17	9	51



4	19	16	76
2	13	4	26
4	16	16	64
30	176	90	479

From the table, we have

$$n = 12, \sum x_i = 30, \sum y_i = 176, \sum x_i^2 = 90, \sum x_i y_i = 479$$

$$\therefore \bar{x} = \frac{\sum x_i}{n} = \frac{30}{12} = 2.5$$

$$\bar{y} = \frac{\sum y_i}{n} = \frac{176}{12} = 14.67$$

$$\begin{aligned} \text{Now, } b_{YX} &= \frac{\sum x_i y_i - n \bar{x} \bar{y}}{\sum x_i^2 - n \bar{x}^2} \\ &= \frac{479 - 12 \times 2.5 \times 14.67}{90 - 12 \times (2.5)^2} \\ &= \frac{479 - 440.1}{90 - 75} = \frac{38.9}{15} = 2.59 \end{aligned}$$



Also,

$$a = \bar{y} - b_{YX} \bar{x}$$

$$= 14.67 - 2.59 \times 2.5 = 14.67 - 6.475 = 8.195$$

$$\therefore a \approx 8.2$$

$\therefore$  The regression equation of percentage increase in weekly sales (Y) on length of weeks (X) is

$$Y = a + b_{YX} X$$

$$\text{i.e., } Y = 8.2 + 2.59 X$$

For  $X = 1.5$ , we get

$$Y = 8.2 + 2.59(1.5) = 8.2 + 3.885 = 12.085$$

$\therefore$  Increase in sales is 12.085% if the campaign has been in progress for 1.5 weeks.

### Miscellaneous Exercise 3 | Q 4.02 | Page 54

The regression equation of y on x is given by  $3x + 2y - 26 = 0$ . Find  $b_{yx}$ .

**Solution:** Given, regression equation of y on x is

$$3x + 2y - 26 = 0$$

$\therefore$  The equation becomes  $2Y = -3X + 26$

$$\text{i.e., } Y = \frac{-3}{2}X + \frac{26}{2}$$

Comparing it with  $Y = b_{YX} X + a$ , we get

$$b_{YX} = \frac{-3}{2}$$



### Miscellaneous Exercise 3 | Q 4.03 | Page 54

If for bivariate data  $\bar{x} = 10$ ,  $\bar{y} = 12$ ,  $v(x) = 9$ ,  $\sigma_y = 4$  and  $r = 0.6$  estimate  $y$ , when  $x = 5$ .

**Solution:**

Given,  $\bar{x} = 10$ ,  $\bar{y} = 12$ ,  $v(x) = 9$ ,  $\sigma_y = 4$ ,  $r = 0.6$

$$\therefore \sigma_x = 3$$

To estimate  $y$ , we should first find the regression equation of  $Y$  on  $X$ .

$$\therefore b_{YX} = r \frac{\sigma_y}{\sigma_x} = 0.6 \times \frac{4}{3} = 0.8$$

$$\text{Also, } a = \bar{y} - b_{YX} \bar{x}$$

$$= 12 - 0.8(10) = 12 - 8 = 4$$

The regression equation of  $Y$  on  $X$  is

$$Y = a + b_{YX} X$$

$$\therefore Y = 4 + 0.8 X$$

For  $X = 5$ ,

$$Y = 4 + 0.8 (5) = 4 + 4 = 8$$

### Miscellaneous Exercise 3 | Q 4.04 | Page 54

The equation of the line of regression of  $y$  on  $x$  is  $y = 2/9 x$  and  $x$  on  $y$  is  $x = y/2 + 7/6$ .

Find (i)  $r$ , (ii)  $\sigma_y^2$  if  $\sigma_x^2 = 4$

**Solution:** Given, regression equation of  $Y$  on  $X$  is

$$y = \frac{2}{9}x$$

$$\text{i.e., } Y = \frac{2}{9}X$$

Comparing with  $Y = b_{YX}X + a$ , we get

$$b_{YX} = \frac{2}{9}$$

and regression equation of X on Y is

$$x = \frac{y}{2} + \frac{7}{6}$$

$$\text{i.e., } X = \frac{1}{2}Y + \frac{7}{6}$$

Comparing it with  $X = b_{XY}Y + a'$ , we get

$$b_{XY} = \frac{1}{2}$$

$$(i) r = \pm \sqrt{b_{XY} \cdot b_{YX}}$$

$$= \pm \sqrt{\frac{1}{2} \cdot \frac{2}{9}} = \pm \sqrt{\frac{1}{9}} = \pm \frac{1}{3}$$

Since  $b_{YX}$  and  $b_{XY}$  both are positive,

$r$  is positive.

$$\therefore r = \frac{1}{3}$$

$$(ii) \text{ Given, } \sigma_X^2 = 4$$

$$\therefore \sigma_X = 2$$

we know that,  $b_{YX} = r \frac{\sigma_Y}{\sigma_X}$

$$\therefore \sigma_Y = \frac{b_{YX} \times \sigma_X}{r} = \frac{\frac{2}{9} \times 2}{\frac{1}{3}} = \frac{4 \times 3}{9} = \frac{4}{3}$$

$$\therefore \sigma_Y^2 = \frac{16}{9}$$

### Miscellaneous Exercise 3 | Q 4.05 | Page 54

Identify the regression equations of x on y and y on x from the following equations,  $2x + 3y = 6$  and  $5x + 7y - 12 = 0$

**Solution:** Given two equations are  
 $2x + 3y = 6$  and  $5x + 7y - 12 = 0$

Let  $2x + 3y = 6$  be the regression equation of Y on X.

$\therefore$  The equation becomes  $2X + 3Y = 6$

$$\text{i.e., } 3Y = 6 - 2X$$

$$\text{i.e., } Y = -\frac{2}{3}X + \frac{6}{3}$$

Comparing it with  $Y = b_{YX} X + a$ , we get

$$b_{YX} = -\frac{2}{3}$$

Now, other equation  $5x + 7y - 12 = 0$  be the regression equation of X on Y.

$\therefore$  The equation becomes  $5X + 7Y - 12 = 0$

$$\text{i.e., } 5X = -7Y + 12$$

$$\therefore X = -\frac{7}{5}Y + \frac{12}{5}$$

Comparing it with  $X = b_{XY} Y + a'$ , we get

$$b_{XY} = -\frac{7}{5}$$

$$\text{Now, } b_{XY} \cdot b_{YX} = -\frac{7}{5} \times \left(-\frac{2}{3}\right) = \frac{14}{15} < 1$$

$\therefore$  Our assumption of regression equation is true.

$\therefore 2x + 3y = 6$  is the regression equation of Y on X, and  $5x + 7y - 12 = 0$  is the regression equation of X on Y.

#### Miscellaneous Exercise 3 | Q 4.06 | Page 54

If for a bivariate data  $b_{yx} = -1.2$  and  $b_{xy} = -0.3$  then find r.

**Solution:** Given,  $b_{yx} = -1.2$  and  $b_{xy} = -0.3$

$$\begin{aligned}\therefore r &= \pm \sqrt{b_{XY} \cdot b_{YX}} \\ &= \pm \sqrt{(-1.2)(-0.3)} = \pm \sqrt{0.36} = \pm 0.6\end{aligned}$$

Since  $b_{XY}$  and  $b_{YX}$  both are negative,

r is negative.

$$\therefore r = -0.6$$

#### Miscellaneous Exercise 3 | Q 4.06 | Page 54

From the two regression equations  $y = 4x - 5$  and  $3x = 2y + 5$ , find  $\bar{x}$  and  $\bar{y}$ .

**Solution:** Given two regression equations are

$$y = 4x - 5$$

$$\text{i.e., } -4x + y = -5 \quad \dots(i)$$

$$\text{and } 3x = 2y + 5$$

$$\text{i.e., } 3x - 2y = 5 \quad \dots(\text{ii})$$

By (i)  $\times 2$  + (ii), we get

$$\begin{array}{r} -8x + 2y = -10 \\ + 3x - 2y = 5 \\ \hline -5x = -5 \end{array}$$

$$\therefore x = 1$$

Substituting  $x = 1$  in (i)

$$-4(1) + y = -5$$

$$\therefore y = 4 - 5 = -1$$

Since the point of intersection of two regression lines is  $(\bar{x}, \bar{y})$ ,

$$\bar{x} = 1 \text{ and } \bar{y} = -1$$

### Miscellaneous Exercise 3 | Q 4.07 | Page 54

The equations of the two lines of regression are  $3x + 2y - 26 = 0$  and  $6x + y - 31 = 0$   
Find

- i. Means of X and Y
- ii. Correlation coefficient between X and Y
- iii. Estimate of Y for  $X = 2$
- iv.  $\text{var}(X)$  if  $\text{var}(Y) = 36$

**Solution: (i)** Given equations of regression are

$$3x + 2y - 26 = 0$$

$$\text{i.e., } 3x + 2y = 26 \quad \dots(\text{i})$$

$$\text{and } 6x + y - 31 = 0$$

$$\text{i.e., } 6x + y = 31 \quad \dots(\text{ii})$$

By (i) - 2  $\times$  (ii), we get

$$3x + 2y = 26$$

$$12x + 2y = 62$$

$$\begin{array}{r} - \quad - \quad - \\ \hline -9x = -36 \end{array}$$

$$\therefore x = \frac{-36}{-9} = 4$$

Substituting  $x = 4$  in (ii), we get

$$6 \times 4 + y = 31$$

$$\therefore 24 + y = 31$$

$$\therefore y = 31 - 24$$

$$\therefore y = 7$$

Since the point of intersection of two regression lines is  $(\bar{x}, \bar{y})$ ,

$\bar{x}$  = mean of  $X = 4$ , and

$\bar{y}$  = mean of  $Y = 7$

**(ii)** Let  $3x + 2y - 26 = 0$  be the regression equation of  $Y$  on  $X$ .

$\therefore$  The equation becomes  $2Y = 3X + 26$

$$\text{i.e., } Y = \frac{-3}{2}X + \frac{26}{2}$$

Comparing it with  $Y = b_{YX}X + a$ , we get

$$b_{YX} = \frac{-3}{2}$$

Now, the other equation  $6x + y - 31 = 0$  is the regression equation of  $X$  on  $Y$ .

$\therefore$  The equation becomes  $6X = -Y + 31$

$$\text{i.e., } X = \frac{-1}{6}Y + \frac{31}{6}$$

Comparing it with  $X = b_{XY}Y + a'$ , we get

$$b_{XY} = \frac{-1}{6}$$

$$\therefore r = \pm \sqrt{b_{XY} \cdot b_{YX}}$$

$$= \pm \sqrt{\frac{-1}{6} \times \frac{-3}{2}} = \pm \sqrt{\frac{1}{4}} = \pm \frac{1}{2} = \pm 0.5$$

Since the values of  $b_{XY}$  and  $b_{YX}$  are negative,

$r$  is also negative.

$$\therefore r = -0.5$$

**(iii)** The regression equation of  $Y$  on  $X$  is

$$Y = \frac{-3}{2}X + \frac{26}{2}$$

For  $X = 2$ , we get

$$Y = \frac{-3}{2} \times 2 + \frac{26}{2} = -3 + 13 = 10$$

**(iv)** Given,  $\text{Var}(Y) = 36$ , i.e.,  $\sigma_Y^2 = 36$

$$\therefore \sigma_Y = 6$$

$$\text{Since } b_{XY} = r \times \frac{\sigma_X}{\sigma_Y}$$



$$\frac{-1}{6} = -0.5 \times \frac{\sigma_X}{6}$$

$$\therefore \sigma_X = \frac{-6}{-6 \times 0.5} = 2$$

$$\therefore \sigma_X^2 = \text{Var}(X) = 4$$

### Miscellaneous Exercise 3 | Q 4.08 | Page 54

Find the line of regression of X on Y for the following data:

$$n = 8, \sum (x_i - \bar{x})^2 = 36, \sum (y_i - \bar{y})^2 = 44, \sum (x_i - \bar{x})(y_i - \bar{y}) = 24$$

**Solution:**

$$\text{Given, } n = 8, \sum (x_i - \bar{x})^2 = 36$$

$$\sum (y_i - \bar{y})^2 = 44, \sum (x_i - \bar{x})(y_i - \bar{y}) = 24$$

$$\therefore b_{XY} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (y_i - \bar{y})^2} = \frac{24}{44} = \frac{6}{11}$$

Now, the regression equation of X on Y is

$$(X - \bar{x}) = b_{XY}(Y - \bar{y})$$

$$\text{i.e., } (X - \bar{x}) = \frac{6}{11}(Y - \bar{y})$$

### Miscellaneous Exercise 3 | Q 4.09 | Page 54

Find the equation of line of regression of Y on X for the following data:

$$n = 8, \sum (x_i - \bar{x})(y_i - \bar{y}) = 120, \bar{x} = 20, \bar{y} = 36, \sigma_x = 2, \sigma_y = 3$$

**Solution:**

$$\text{Given, } n = 8, \sum (x_i - \bar{x})(y_i - \bar{y}) = 120,$$

$$\bar{x} = 20, \bar{y} = 36, \sigma_x = 2, \sigma_y = 3$$

$$\therefore \text{Var}(X) = \sigma_X^2 = 4$$

$$\text{Since Var}(X) = \frac{\sum(x_i - \bar{x})^2}{n},$$

$$4 = \frac{(\sum(x_i - \bar{x}))^2}{8}$$

$$\therefore \sum(x_i - \bar{x})^2 = 32$$

$$\text{Now, } b_{YX} = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sum(x_i - \bar{x})^2} = \frac{120}{32} = 3.75$$

$\therefore$  The regression equation of Y on X is

$$(Y - \bar{y}) = b_{YX}(X - \bar{x})$$

$$\therefore (Y - 36) = 3.75(X - 20)$$

$$\therefore Y - 36 = 3.75X - 75$$

$$\therefore Y = 3.75X - 75 + 36$$

$$\therefore Y = 3.75X - 39$$

### Miscellaneous Exercise 3 | Q 4.1 | Page 54

The following results were obtained from records of age (X) and systolic blood pressure (Y) of a group of 10 men.

	X	Y
Mean	50	140
Variance	150	165

$$\text{and } \sum(x_i - \bar{x})(y_i - \bar{y}) = 1120$$

Find the prediction of blood pressure of a man of age 40 years.

**Solution:** Given, X = Age, Y = Systolic blood pressure,

$$n = 10, \bar{x} = 50, \bar{y} = 140,$$

$$\sigma_X^2 = 150, \sigma_Y^2 = 165 \text{ and}$$

$$\sum (x_i - \bar{x})(y_i - \bar{y}) = 1120$$

$$\text{Since Var}(X) = \frac{\sum (x_i - \bar{x})^2}{n},$$

$$\sigma_x^2 = \frac{\sum (x_i - \bar{x})^2}{n}$$

$$\therefore 150 = \frac{\sum (x_i - \bar{x})^2}{10}$$

$$\therefore \sum (x_i - \bar{x})^2 = 1500$$

$$\text{Now, } b_{YX} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2} = \frac{1120}{1500} = 0.7$$

$\therefore$  The regression equation of systolic blood pressure of the men (Y) on their age (X) is

$$(Y - \bar{y}) = b_{YX}(X - \bar{x})$$

$$\therefore (Y - 140) = 0.7(X - 50)$$

$$\therefore Y - 140 = 0.7X - 35$$

$$\therefore Y = 0.7X - 35 + 140$$

$$\therefore Y = 0.7X + 105$$

For X = 40,

$$Y = 0.7(40) + 105 = 28 + 105 = 133$$

$\therefore$  The man of age 40 years has systolic blood pressure 133.

**Miscellaneous Exercise 3 | Q 4.11 | Page 54**

The equations of two regression lines are  $10x - 4y = 80$  and  $10y - 9x = -40$  Find:

i.  $\bar{x}$  and  $\bar{y}$

ii.  $b_{YX}$  and  $b_{XY}$

iii. If  $\text{var}(Y) = 36$ , obtain  $\text{var}(X)$

iv.  $r$

**Solution: (i)** Given equations of regression are

$$10x - 4y = 80$$

$$\text{i.e., } 5x - 2y = 40 \quad \dots(i)$$

$$\text{and } 10y - 9x = -40$$

$$\text{i.e., } -9x + 10y = -40 \quad \dots(ii)$$

By  $5 \times (i) + (ii)$ , we get

$$25x - 10y = 200$$

$$\begin{array}{r} -9x + 10y = -40 \\ \hline 16x = 160 \end{array}$$

$$\therefore x = 10$$

Substituting  $x = 10$  in (i), we get

$$5(10) - 2y = 40$$

$$\therefore 50 - 2y = 40$$

$$\therefore -2y = 40 - 50$$

$$\therefore -2y = -10$$

$$\therefore y = 5$$

Since the point of intersection of two regression lines is  $(\bar{x}, \bar{y})$ ,  
 $\bar{x} = 10$  and  $\bar{y} = 5$

**(ii)** Let  $10y - 9x = -40$  be the regression equation of Y on X.

$\therefore$  The equation becomes  $10Y = 9X - 40$

$$\text{i.e., } Y = \frac{9}{10}X - \frac{40}{10}$$

Comparing it with  $Y = b_{YX}X + a$ , we get

$$b_{YX} = \frac{9}{10} = 0.9$$

Now, the other equation  $10x - 4y = 80$  be the regression equation of X on Y.

$\therefore$  The equation becomes  $10X = 4Y + 80$

$$\text{i.e., } X = \frac{4}{10}Y + \frac{80}{10}$$

$$\text{i.e., } X = \frac{2}{5}Y + 8$$

Comparing it with  $X = b_{XY}Y + a'$ , we get

$$b_{XY} = \frac{2}{5} = 0.4$$

**(iii)** Given,  $\text{Var}(Y) = 36$ , i.e.,  $\sigma_Y^2 = 36$

$$\therefore \sigma_Y = 6$$

$$\text{Since } b_{XY} = r \times \frac{\sigma_X}{\sigma_Y}$$

$$\frac{2}{5} = 0.6 \times \frac{\sigma_X}{6}$$

$$\therefore \frac{2}{5} = 0.1 \times \sigma_X$$

$$\therefore \frac{2}{5 \times 0.1} = \sigma_X$$

$$\therefore \sigma_X = 4$$

$$\therefore \sigma_X^2 = 16 \text{ i.e., Var}(X) = 16$$

$$\text{(iv) } r = \pm \sqrt{b_{XY} \cdot b_{YX}}$$

$$= \pm \sqrt{\frac{2}{5} \times \frac{9}{10}} \pm \sqrt{\frac{9}{25}} = \pm \frac{3}{5} = \pm 0.6$$

Since  $b_{YX}$  and  $b_{XY}$  are positive,

$r$  is also positive.

$$\therefore r = 0.6$$

### Miscellaneous Exercise 3 | Q 4.12 | Page 54

If  $b_{YX} = -0.6$  and  $b_{XY} = -0.216$ , then find correlation coefficient between  $X$  and  $Y$ .  
Comment on it.

**Solution:** Given,  $b_{YX} = -0.6$ ,  $b_{XY} = -0.216$

$$\therefore r = \pm \sqrt{b_{XY} \cdot b_{YX}}$$

$$= \pm \sqrt{-0.216 \cdot (-0.6)} = \pm \sqrt{0.1296}$$

$$\therefore r = \pm 0.36$$

Since  $b_{XY}$  and  $b_{YX}$  are negative,

$r$  is also negative.

$$\therefore r = -0.36$$

$\therefore X$  and  $Y$  negatively correlated.